Topological Dynamics in the Physical and Biological Sciences

16 July-21 December 2012

Report from the Organisers

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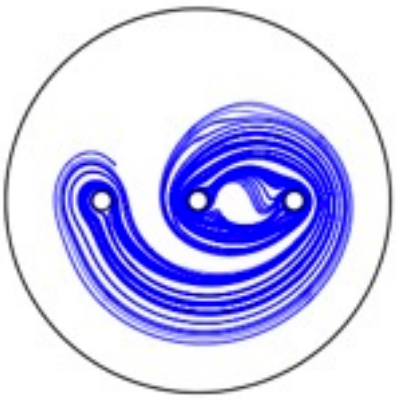
The motivation for this programme was described in the following terms:

The programme is intended to stimulate interaction between applied mathematicians, biologists and physicists who frequently encounter dynamical problems that have some explicit or implicit topological content. We use the term 'topological' to convey the idea of structures, e.g. knots, links or braids in 3D, that exhibit some measure of invariance under continuous deformation. Dynamical evolution is then subject to the topological constraints that express this invariance. A basic common problem is to determine minimum energy structures (and routes towards these structures) permitted by such constraints; and to explore mechanisms, e.g. diffusive, by which such constraints may be broken.

The programme was planned around four themes exploring fields in which topological constraints can be of critical importance: vortex dynamics; DNA function and protein folding; MHD in astro- and plasma physics; and quantized-flux systems. We were fortunate to have a good number of participants who stayed for at least four months of the programme, helping to give it an excellent degree of coherence and continuity. We held two seminars each week, and a number of 'mini-talks' and discussions proposed by individual participants. We endeavoured to avoid overloading the programme with too many seminars, in order to allow ample time for reflection and for new ideas to develop through informal discussion. The workshops and seminars were streamed live and are accessible at http://www.newton.ac.uk/programmes/TOD/seminars/.

Theme 1: Vortex Dynamics

The programme started with a Symposium sponsored by the International Union of Theoretical and Applied Mechanics (IUTAM). The Proceedings have been published online (open access) at www.sciencedirect.com science/journal/22109838/7 by Elsevier, under the title: *Topological Fluid Dynamics; Theory and Applications,* and will soon also be available in hardcopy. This Symposium introduced topics that provoked sustained discussion throughout the programme. An *Aref Memorial Lecture* was given by M Brøns in tribute to Hassan Aref (Virginia Tech), who had served on the Scientific Committee for the Symposium until his sudden and tragic death in September 2011. This lecture was concerned with recent advances made by Aref and his colleagues on the very classical problem of equilibrium (rotating) configurations of point vortices. Further highlights were lectures by A Enciso and D Peralta-Salas, who have proved the remarkable result that, given an arbitrary knot, there exists an analytic Beltrami flow (vorticity everywhere parallel to velocity) for



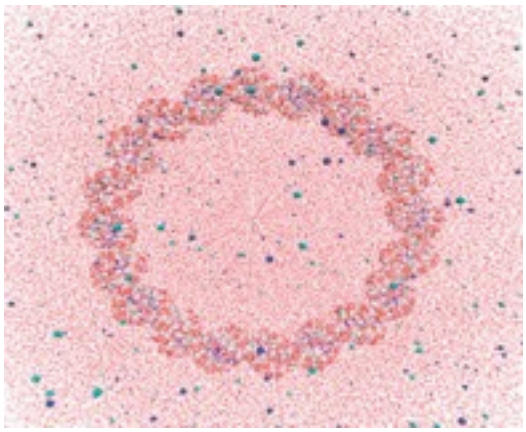
Material line pattern for a three-braid 'pseudo-Anosov' mixer after three periods (courtesy S.E.Tumasz & J.-L.Thiffeault).

which this knot is a vortex line; and lectures by P Boyland, M Stremler, and J-L Thiffeault, on their pioneering work (partly in collaboration with Aref) on the application of Thurston-Nielsen theory to the problem of stirring a fluid by the periodic movement of three or more stirring rods. This stimulated much fruitful

interaction between the pure and applied participants of the programme.

Theme 2: DNA Function and Protein Folding

This theme involved close collaboration between A Bates, D Buck, S Harris, A Stasiak, and DW Sumners, who orchestrated lectures and discussion on the modelling of DNA molecules subject to topological constraints, the action of DNA topoisomerases, DNA recombination and its mechanisms,



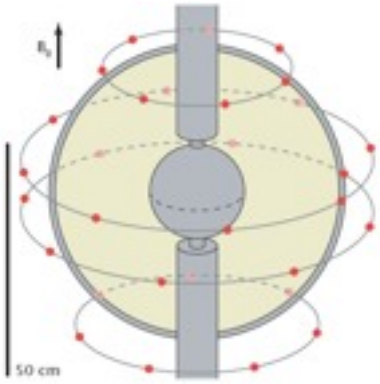
An explicitly solvated starting structure (110 bp, 0.1 mol NaCl) showing DNA and counter ions in a space-filling representation (courtesy J.S.Mitchell, C.A.Laughton & S.A.Harris, Nucl.Acids Res.2011).

chromosomal architecture, folding mechanisms of knotted proteins, and the function of knots in proteins. The associated workshop attracted 111 participants, with 48 talks and 21 posters. The journal Nucleic Acids Research sponsored two prizes for outstanding student posters, with winners T Sutthibutpong (Leeds) and K Valencia (Imperial College London). Topical mini-reviews from the workshop are published in the April 2013 issue of *Biochemical Society Proceedings*.

An **Open-for-Business** event *Maths meets Molecular Biology at the Newton Institute* was held on 16 September, at which C Dobson, Master of St John's College, lectured on *New approaches to understanding and treating neurodegenerative diseases*, and Sir G Winter, Master of Trinity College, lectured on *The business of science; building therapeutic drugs based on proteins*. Also noteworthy was the lecture of L Zechiedrich (Baylor College of Medicine, Houston, Texas) on how basic DNA topology research has opened doors for gene therapy. D Buck and DW Sumners opened this event with impassioned presentations on the relevance of topological concepts in understanding sub-cellular biological processes, topics central to the discussions and interactions that took place subsequently in the programme.

Theme 3: MHD in astro- and plasma physics

This theme involved sustained interactions between C Parnell, K Bajer, Y Kimura, K Moffatt and G Hornig, with particular focus on braiding of magnetic fields and formation of current sheet discontinuities. A satellite workshop was held at ICMS, Edinburgh, at which lectures by A Title (Lockheed) showing maps of solar magnetic field topology and evolution, and by D Lathrop (Univ. of Maryland) on his experiments in metallic sodium and liquid helium, were particularly notable.



Spherical annulus 'Taylor-Couette' experiment; liquid sodium fills the spherical annulus, an axial magnetic field is applied, and the boundary spheres may be independently rotated. Hall probes are placed at 30 points as indicated (courtesy DA Lathrop, Phys.Rev.E 2010).

On the Tuesday evening, a Reception was held at the Royal Society of Edinburgh in George Street, where, by lucky chance, an exhibition on the development of theoretical physics

"from Maxwell to Higgs" was on display and much appreciated by the participants.

During November, the focus turned to problems involving surface topology, involving collaboration among M.Monastyrski (Moscow), E Panagiotou (TU, Athens), E.Kats (Landau Institute), V.M.Buchstaber (Steklov Institute) , R.Kerner (Paris VI), R.Kusner (Univ. of Massachusetts), A.Mal'tsev (Landau Institute), S.Rouhani (Sharif Univ. of Tech., Tehran) and RE Goldstein (Cambridge). P Wadhams and T Wagner (DAMTP) gave a stimulating presentation on aspects of iceberg dynamics, covering experiments carried out during the expedition to northern Greenland as described in the BBC2 Film *Operation Iceberg*.

A soap film spanning a twisted wire loop that makes two complete turns can take the form of a Mobius strip. Untwisting the wire leads the soap film to jump from one-sided to two-sided form. A twist localised near the wire boundary persists after the jump, as shown in this image (courtesy RE Goldstein, HK Moffatt, AIPesci (Cambridge) & RLRicca (Milano-Bicocca).

Theme 4: Quantized-flux systems

The concept of knotted and linked quantized flux was a recurrent theme, culminating in a December workshop organized by N Berloff , J Cantarella , A-C Davis, T Kephart, P Sutcliffe and T Vachaspati (100 participants from 20 countries; 64 talks, 7 posters). In addition, Sir Michael Berry (Bristol) gave the Rothschild lecture entitled *Superoscillations and weak measurement*.

A tight knot (7_2) in the geometrical configuration that minimizes the ratio of ropelength to diameter (courtesy J Cantarella).

Throughout the programme, there was much interaction between K Millett, J Cantarella, C Shonkwiler, R Kusner, and others on the fundamental issues of the energy spectrum of knots and links, of tightening, of relaxation to local and global minima, and of topological aspects of stability related to helicity. The final workshop turned to quantized helicity and its generalizations, curvature, distortion and other physical corrections, with application to quantum reconnection and tunneling, monopole-anti-monopole pair production, and universality aspects of tightly knotted systems of quantized flux, from super-conductors to glueballs in QCD.

A combined proceedings of the 3rd and 4th workshops is in preparation and will be published with online open access by the Institute of Physics.

The Seminar Programme

The 11.30 a.m. seminars on Tuesdays and Thursdays, starting with P Boyland's *Some topological tools in two-dimensional dynamics* on 19 July, and ending with S Nazarenko's *Quadratic invariants for clusters of resonant wave triads* on 20 Dec., provided ample stimulation and input to the animated discussions that followed over lunches in

Wolfson Court and after-lunch coffee at the Institute. To mention just a few that give a good impression of the scope of the programme: A Herczynski gave a thought-provoking seminar (7 Aug.) on *painting with viscous jets*, a retrospective analysis of the painting technique of the artist Jackson Pollock, inspired by the sinusoidal tracks evident on his canvases; W Irvine (11 Sept.) gave an intriguing discussion of his experimental creation of a knotted vortex and the manner in which it reconnects to form two unlinked vortex rings; C Parnell (4 Oct.) gave a beautiful presentation of the topology of complex magnetic fields of the Sun and the magnetosphere; and T Kephart (6 Nov.) gave his updated theory (with R Buniy) on the tight knot spectrum of glueballs in quantum chromodynamics.

In conclusion, we believe that the programme succeeded in its aim of promoting interactive collaboration between mathematicians and physical and biological scientists, and raised problems that will continue to challenge in the years to come. A follow-up short meeting in 3--5 years' time would be valuable in order to assess the impact of the programme.

The Newton Institute provided a perfect setting for a sustained research programme of this kind. We are immensely grateful to the Staff of the Institute for their vital support at all stages, and to the Director and Scientific Steering Committee for the trust placed in us to run this ambitious and wide-ranging programme.