Newton Institute Satellite Programme

Topology, representation theory and higher structures

26 May 2024 to 22 June 2024

Final Report

The past decades have seen a surge of multidisciplinary mathematical activity on which the programme "Topology, representation theory and higher structures" aimed to capitalise. Emerging new techniques have allowed researchers to solve long-standing problems such as the Kervaire invariant problem or higher-dimensional versions of the Madsen–Weiss theorem. At the same time, there have been profound breakthroughs in manifold topology such as the computations of homotopy types of cobordism categories, the immense success of gauge theory and ideas from physics in manifold topology, and the development of quantum invariants in the context of representation theory and quantum groups. In each of these areas, the growing importance of higher structures underscored the need for a dedicated programme to explore these connections.

The programme was designed to bring together established researchers in homotopy theory, geometry, representation theory, quantum algebra, and higher categories. Facilitating collaboration among these diverse fields, the programme aimed to push the boundaries of current knowledge and foster the development of new approaches.

The programme was run as a 4-week satellite event at the Sabhal Mòr Ostaig, ordinarily a college for Gaelic language, set against the backdrop of the beautiful scenery of the Isle of Skye. The secluded setting of the conference venue allowed participants to concentrate on their mathematical research and to fully immerse themselves in mathematical discussions and potential future collaborations. The staff at the college were exceptional and took great care to ensure the event ran smoothly, and their hospitality was greatly appreciated by the participants. The college's inclusive spaces encouraged collaboration and fostered an atmosphere of creativity.

The programme was organized into four different weeks to maximize its impact. The first two weeks were dedicated to collaborative research, providing a platform for up to 40 invited researchers to engage in in-depth discussions and exchange ideas. The third week was a workshop tailored to graduate students and ECRs and featured mini courses from invited experts. This week aimed to equip emerging mathematicians with the foundational knowledge and tools needed to contribute to the field's ongoing developments. The programme culminated in a full-scale international conference in the fourth week, which drew together leading experts and participants to discuss the future research directions in all the programme's areas. The programme attracted many

PhD students and postdocs, and most of them delivered a talk about their research during their stay.

Some of the challenges that were discussed during the programme are as follows.

- The study of tensor categories is fundamental to the understanding of topological quantum field theories and many areas in mathematical physics. The classification of tensor categories is a widely open problem, which can be considered as a wide generalisation of the classification problem for finite groups.
- In geometric group theory, topics of recent interest include the K(π,1) conjecture, realisation problems, representation stability and new functorial approaches to the representation theory of classical groups such as braid groups and mapping class groups of manifolds in dimensions 3 and 4.
- Open questions in unstable homotopy theory and related topics which would benefit from new ideas originating from higher category theory. For instance, spaces of self homotopy equivalences of p-local finite groups are well understood, but the nature of general mapping spaces between p-local groups is a wide-open problem.
- Many results in manifold topology are currently limited to surfaces and remain unexplored in higher dimensions. New developments can be expected to draw on advances in higher category theory to describe and exploit refined notions of symmetry.

Scientific Outcomes

The programme was designed to provide maximum flexibility for participants to advance their research, while simultaneously learning new mathematics and discovering potential new research directions and collaborations. To achieve this, the first two weeks were dedicated to free research collaboration with a single talk per day. The third week was a workshop on the four different themes of the programme. Week 4 was an international conference. By dividing the 4-week period in this way, we created an environment rich in structure and possibilities, designed to foster and promote new collaborations and research directions. A testimonial to the success of the programme can be found in the response to the Scientific Survey sent out by Newton Institute, where 87.5% rated the programme as 'Excellent' and the rest of them as 'Good'.

Early career researchers (ECR): A notable highlight of the programme was the inclusion of many early career participants. We hosted a significant number of advanced PhD students and postdocs, which contributed immensely to making the event very dynamic and interesting, since they were very actively engaged. Particularly the younger participants benefited from being exposed to the very wide mathematical scope our programme had. Among the 93 participants in total that attended the programme, 22 were female. Thanks to careful planning we managed to accommodate a short talk by every ECR who wished to present. Most of these talks were brilliantly delivered and very well received.

Research in groups: The first two weeks of the programme were intended for groups of researchers to get together and work on existing collaborative projects, as well as explore new projects. During these two weeks we limited presentations to one per day or occasionally two shorter talks. These were intended to give structure to the day of the participants and to be a starting point for engaging with the other participants. Around 30 participants were present during this time. This part of the programme was very well received, as one participant noted "I think the flexibility given to the attendees was great. Limiting the contributed talks to one a day was great, though it was good to have at least one talk." The last comment refers to the fact that whoever wished to give a talk was afforded the opportunity to do so.

Mini-courses: The third week of our programme consisted of five mini courses, delivered in the form of two-hourly morning talks by experts in their respective fields. The mini courses were delivered assuming minimal prior knowledge of the area, and also gave ample references to the literature. The recordings of the mini courses were posted promtly, facilitating learning during the programme. These were followed in the afternoon by shorter contributed talks in topics related to the talks in the morning. This part of the programme was intended specifically for early career participants, and indeed was very popular among them. Indeed, we received a very large number of applications weeks 3 and 4 of our programme. Very lively discussions followed presentations. One participant commented: "The organizers set up a perfect program structure. The participants were very carefully selected which resulted in an unusual high number of long discussions."

International Conference: Week 4 of the programme was an international conference, themed around the topics of the programme. We had two plenary talks every day by distinguished senior mathematicians as well as several excellent younger researchers. The afternoon sessions again consisted of shorter contributed talks. The conference was run as a hybrid event for the plenary talks, three of which were delivered by live streaming. The equipment provided by INI allowed this to run very smoothly. Although we had not planned any hybrid component in our program, three of our invited speakers informed us at a late stage that they had some hurdles preventing them to come in person. We were very grateful that the INI provided the equipment to face this challenge so that the talks could be delivered online. This way of delivery naturally turned out to be less inspiring than on-site talks, since communication between audience and speaker was quite limited for technical reasons. The selection of plenary speakers was done in accordance with the intended themes of the programme, and it had the required effect. As one participant commented, "The plenary speaker selection also provided a very good range of seniorities and backgrounds".

Interactions and Collaborations: The programme was designed to host mathematicians from four different sub-disciplines, close enough so that researchers would be able to understand a good deal of each other's work, but far enough to foster new interests and collaborations. This aspect of the programme worked extremely well as indicated by lively discussions that followed almost every talk. Interactions among different topics

benefited particularly from the presence of many early career researchers, who showed great initiative to learn and discuss new material and build bridges between their own work and that of others. Among respondents to the INI Scientific survey 30% testified to having started a new collaboration as a result of participation in the programme.

Advancement of Research: A previous conference held in the same location, involving both algebraic and geometric topology, proved to be very diverse and successful. Our satellite programme was inspired in particular by this event, but since we had funding for four weeks, we designed the programme to give ample opportunity for researchers from different mathematical sub-disciplines to meet, discuss work and attempt to move out of their comfort zone toward potential new fields and collaborations. This made our programme very unique in the sense that it involved participants from four mathematical areas that would normally not talk directly to each other. One of our aims was to break this over-specialisation pattern.

The effort proved to be very successful. In response to the question to participants on whether the programme opened new research directions for them 37.5% responded `Yes', and additional 50% answered `Partially'. This may have been one of the most important aspects of the programme, as it gave an opportunity for researchers to meet other mathematicians that they would not normally encounter in their own research area. While one cannot expect major work to have been produced within a 4-week programme, it is clear that the impact of the programme will last long after its end in giving new insights and collaborative opportunities to participants.

Future: It is impossible to predict what the future holds in terms of new directions that arise from communication among mathematicians in our programme. One may nonetheless predict that new work will arise as a result of contacts made during the programme. Furthermore, in the many reactions conveyed to the organisers by many participants, the programme created an appetite for more events of this type. Indeed, one of the most common questions we were asked was "when is the next one planned?".