Final report on the Newton Insitute Programme

THEORY OF WATER WAVES

14 July – 8 August 2014

Organisers: T.J. Bridges, M.D. Groves, P.A. Milewski, & D.P. Nicholls[1]

1 Background and Historical Information

Waves on the surface of the ocean are a dramatic and beautiful phenomena that impact every aspect of life on the planet. At small length scales, ripples driven by surface tension on the surface of these "water waves" affect remote sensing of underwater obstacles. At intermediate scales, waves on the surface and the interface between internal layers of water of differing densities affect shipping, coastal morphology, and near–shore navigation. At larger lengths, tsunamis and hurricane–generated waves can cause devastation on a global scale. Additionally, water waves play a crucial role at all length scales in the exchange of momentum and thermal energy between the ocean and atmosphere which, in turn, affect the *global* weather system and climate.

From a mathematical viewpoint, the water wave equations pose severe challenges for rigorous analysis, modeling, and numerical simulation. The governing equations are widely accepted and there has been substantial research into their validity. However, a rigorous theory of their solutions is extremely complex due not only to the fact that the water wave problem

[1]Principal Organiser

is a classical free boundary problem, where the domain shape is unknown, but also because the boundary conditions are strongly nonlinear. The level of difficulty is such that the theory has merely begun to answer the fundamental questions which *must* be addressed before our understanding can be termed "adequate." For instance, it is well known that the very *existence* of solutions to the equations that describe fluid motion, even in the absence of free boundaries, is one of the most difficult unanswered questions in mathematics (indeed it is one of the famed Clay Math Institute Millennium Prize Problems).

Although water waves have intrigued mankind for thousands of years, it was not until the middle of the nineteenth century that the modern theory appeared, principally in the work of Stokes. The nineteenth century also produced useful models for tidal waves, solitary waves, the Korteweg–de Vries (KdV) equation, the Boussinesq models for shallow water waves, the Kelvin–Helmholtz instability, Cauchy–Poisson circular waves, Gerstner's rotational waves, Stokes' model for the highest wave, and Kelvin's model for ship wakes.

The twentieth century has witnessed even more substantial advances in the theoretical understanding of nonlinear water waves and the list of major advances is too numerous to give here. However, advances relevant to this programme are: Wilton ripples and capillary–gravity effects, Crapper waves and other capillary phenomena, rigorous theories for a large range of two–dimensional waves, the Benjamin–Feir instability, new models such as the Kadomtsev–Petviashvili (KP) and Nonlinear Schrödinger (NLS) equations, the superharmonic instability, wave–interaction theory and weak turbulence, the discovery of the Hamiltonian and Lagrangian structures for water waves, the discovery of new solitary waves, and rigorous theories for three–dimensional periodic traveling waves and localized patterns. Central to many of these discoveries was the invention and implementation of ever more efficient and robust numerical methods for steady waves, the initial value problem, and the spectral stability problem.

1.1 The outstanding problems going into the programme

On a general level, outstanding areas of interest going into the programme were: the need for new model equations, new analytical techniques, stability theories, numerical methods for both steady and unsteady waves, and the extension of all theories into three-dimensionality and time dependence.

1. While a whole host of nonlinear model Partial Differential Equations (PDEs) valid in various scaling regimes are now at our disposal, more model equations for water waves are needed. For instance, many model equations are inadequate to capture effects of (i) full three dimensionality, (ii) capillarity, (iii) shorter wavelengths, (iv) larger amplitudes, (v) irrotationality, and (vi) weak viscosity. New models will provide a simplified setting for analyzing qualitative properties of such effects.

2. Of course, new models will require new analytical techniques; for instance, the legendary complete integrability properties of some classical models (e.g., KdV, NLS, KP) will no longer be available and possibly the greatest challenge we will face is the development of new methodologies for *non*–integrable models of water waves.

3. One crucial question facing all developments, both analytical and numerical, in the theory of water waves is that of stability: Once one has found a solution of interest will it be observable in the laboratory setting? Will it be found in the open ocean?

4. There are now a range of rigorous mathematical results for two–dimensional periodic waves such as the Benjamin–Feir and superharmonic instabilities found in periodic wavetrains

and solitary waves on deep water. However there are major open questions about not only existence but also dynamic stability of genuinely three–dimensional patterns. For example: experiments and oceanographic observation show that surface water waves with hexagonally–shaped surface exist and are robust three–dimensional patterns in *shallow* water, however, no rigorous stability theory has been proposed to study this class of patterns. Additionally, what is their stability in deep water? What about non–hexagonal waveforms?

On a specific level, questions that were raised by the participants going into the programme included: stability questions (Akers, Deconinck, Groves, Hur, Kalisch, Kataoka, Nicholls, Oliveras, Trichtchenko, Wahlen), numerical methods (Akers, Clamond, Fuhrman, Gagarina, Nicholls, Wilkening), wave breaking and singularities (Bredmose, Cooker, Grue, Shkoller, Wu), resonances (Akers, Kataoka, Tobisch), spatial dynamics (Groves, Wahlen), 2D and 3D solitary waves (Akylas, Groves, Milewski, Wahlen, Vanden-broeck, Wahlen, Zhang), modulation theory (Bridges), wave-current interaction (Shrira), normal forms (Craig), waves with vorticity (Groves, Kharif, Varvaruca, Wahlen), bottom topography (Nachbin), 3D standing waves (Plotnikov, Wilkening), shallow water hydrodynamics (Bokhove, Bridges, Deconinck, Gagarina, Grimshaw, Kalisch, Segur), well-posedness and validity (Ambrose, Nicholls, Schneider, Totz, Wu), non-local formulations (Ashton, Fokas), wind-wave interaction (Hunt, Janssen, Onorato, Sajjadi, Shrira).

2 Programme Timeliness, Scope and Outline

The programme was timely for three reasons: it follows on from the 2001 Newton Institute Programme on "Surface Water Waves" [4], it captures the recent momentum in the subject which has seen an expansion in the rigorous analysis and numerical analysis of water waves, and it resonated with the SIAM Nonlinear Waves conference [6] which was held immediately afterwards.

The programme was an international event with participants from Australia, Austria, Brazil, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Norway, Russia, Sweden, UK, USA, with the majority of participants (35) coming from the UK and USA.

Outside of the event days, the conference was very loosely organized, and as the participants began interacting a "style" emerged. The programme settled into a pattern of morning lectures followed by open afternoons filled with one–on–one and small–group interactions. Due to the myriad challenges and wide range of specializations and expertise required to address them, this approach emerged as an efficient way of proceeding. Indeed, one of the most interesting outcomes of this strategy was the enthusiastic interaction between researchers working primarily on physical aspects and methodological approaches and those working on pure mathematics and strictly theoretical constructs.

2.1 Brief summary of workshop events

The key events in Week 1 were

- **Opening Day**: where distinguished leaders in the field set the agenda by presenting recent results and open problems on water waves. The speakers were Janssen, Shrira, Segur, Akylas, Craig, and Plotnikov. (Organised by Nicholls.)
- **Shortcourse**: four hours on Higher Order Perturbation of Surfaces (HOPS) methods which combine analysis and numerics to study existence and stability of water waves

(Akers & Nicholls), and three hours on the unified transform and non-local formulations for water waves (Fokas). (Organised by Nicholls.)

- **Numerics Days**: where 10 hours of talks on a range of numerical methods for water waves were presented over two days. The speakers were Parau, Wang, Vanden-Broeck, Wilkening, Trichtchenko, Oliveras, Deconinck, Guyenne, Clamond, and Gagarina) (Organised by Nicholls.)

- **Microsymposium on Wind-Wave interaction**: two hours by Hunt & Sajjadi on recent results on wind-wave interaction with particular emphasis on the role of grouping of waves and CFD simulations. (Organised by Bridges.)

Spitalfields Day: in Week 2 the main event was the London Mathematical Society Spitalfields Day, organised by Groves. A report on this event has appeared in the LMS Newsletter and is available electronically [1].

In Week 3, there were two planned events.

- **Water Waves in Industry Day**: this event was sponsored by the Turing Gateway and organised by Bridges with key support from Jane Leeks and Clare Merritt at Turing Gateway. It brought in speakers and participants from industry to discuss their interaction with the theory of water waves. The speakers included Dias (UCD), Ken Doherty (Aquamarine Power), Nigel Bunn (HR Wallingford), Hvoje Jasak (Wikki Ltd), Rod Rainey (Atkins Oil & Gas), and a Discussion Session chaired by Tom Bridges.

- **Quantum Mechanics & Water Waves Event**: this event was organised by Milewski. It brought together the two Newton programmes running at that time (the other being a programme on quantum control). A talk was given by Paul Milewski to both programmes showing how droplets on the surface of a liquid could behave similarly to particles in quantum mechanics. There was a followup talk the next week by Robert Brady and the video of his talk is the most popular download of the entire water waves programme.

Summer School: in Week 4 the main planned event was a Summer School on Water Waves organised by Bridges with key support from Almarie Williams. It had 10 hours of lectures over 3 days given by 5 lecturers (Ambrose, Bokhove, Bridges, Nachbin, Wu). The school attracted about 30 participants from the UK, Europe, and US.

2.2 Brief summary of SIAM Nonlinear Waves satellite meeting

The programme dovetailed with the bi-annual meeting of the Society for Industrial and Applied Mathematics (SIAM) activity group on "Nonlinear Waves and Coherent Structures", which took place at Churchill College during 11–14 August 2014. The conference attracted over 400 participants from around the world. WW2014 organiser Paul Milewski was one of the Principal Organisers of this event. Many of the WW2014 participants also attended this meeting, and there was a wide range of talks on water waves [6].

3 Scientific Outcomes and Highlights

The programme was so short that new ideas that emerged during the programme are still developing. However, there were surprises during the programme that did impact the way the participants thought about the theory of water waves. Some of the highlights that had a high impact were the talks:

- New theory for jet currents, explaining the large waves associated with the Agulhas current (Shrira)
- Wide range of talks on 2D and 3D solitary waves (Akylas, Groves, Parau, Vanden-Broeck, Wahlen, Zhang)
- Prediction of a dynamic cascade phenomena generated by a Benjamin-Feir instability (Tobisch)
- High performance numerics for 2D and 3D large amplitude periodic standing waves (Wilkening)
- New results on self-similar waves with acute crest angles (Wu).
- The wide range of numerical methods being developed and applied to the time-dependent problem (Akers, Bokhove, Gagarina, Guyenne, Nicholls)
- New results on the stability of water waves (Akers, Hur, Oliveras, Trichtchenko, Wahlen)
- New rigorous, numerical and experimental results on breaking waves, splashing, and singularities (Bredmose, Grue, Shkoller)
- Extension of validity results for NLS to three dimensions using Clifford analysis (Totz)
- Rigorous results on large amplitude travelling waves with vorticity (Varvaruca, Wheeler)
- A success story of the interaction between academia and industry in the area of ocean wave energy harvesting (Dias, Doherty)
- A connection between quantum mechanics and free surface flows (Brady, Milewski)

3.1 The forward-looking session

The programme held a forward looking session on Wednesday 6th August, chaired by Tom Bridges, Mark Groves and Paul Milewski. The session started with a backward looking session, reviewing and discussing the topics covered during the programme. A lively discussion was held on the relative merits of the various results and approaches. It was agreed that the programme, and the present state of the art, barely scratched the surface of the wide range of problems of interest in the theory of water waves. Topics and problems flagged up for future work include: post-instability dynamics; the importance of well-posedness particularly for truncated models; the need for global well-posedness to validate nonlinear stability of solitary waves; what models and equations are appropriate for breaking, splashing, entrainment, and bubbles; a hierarchy of models and formulations; a encyclopedia of known results (analytical, numerical, rigorous) on all steady and time-periodic waves and their stability; new numerical methods and comparison between different methods; any theoretical development on (a) wind-wave interaction, (b) three dimensional waves of every shape and form, (c) extreme waves, and (d) any type of breaking waves.

3.2 How the programme will advance research in the field

Due to the shortness of the programme it is difficult to measure advancement during the programme. However, it has been set up for long lasting impact. The fantastic video programme of the Newton Institute, where over 50 of the lectures were filmed and will be freely available in perpetuity, will expand the impact. Similarly, the blog [3] contains detail on talks, the demonstration, anecdotal commentary and a summary of the forward looking session that is also freely available in perpetuity.

4 Multimedia Activity and Publication

4.1 The Blog

A blog [3] was run during the programme by Tom Bridges. The idea was to record informal descriptions of the talks, some of the discussions, Bokhove's experiment, and the forward looking session. It provides a useful resource for particpants wishing to remind themselves about the range of talks, and for interested researchers around the world to get an impression of the atmosphere at the programme.

4.2 Evidence of remote access to streamed lectures

Most of the lectures were recorded are are available on the permanent Cambridge University website at [2]. They continue to be downloaded on a regular basis. The top ten downloaded presentations as of 16/10/2014 are

- Brady (quantum droplets on a free surface) 291
- Akers & Nicholls (HOPS shortcourse, all 4 lectures) 182
- Fokas (unified transform shortcourse, all 3 lectures) 172
- Milewski (quantum mechanics and water waves) 140
- Wahlen (transverse instability of solitary waves) 138
- Guyenne (numerical methods for water waves) 112
- Varvaruca (global bifurcation with vorticity) 99
- Colman (statistics of the 10000 year wave) 99
- Bunn (modelling at HR Wallingford) 96
- Groves (three-dimensional waves) 91

4.3 Lectures delivered over the internet from other locations

As part of the Water Waves in Industry Day, Dr Ken Doherty of Aquamarine Power, gave a talk from his desk in Belfast, Ireland, which was broadcast live at the event, including immediate feedback from the audience. His talk followed a talk by Frederic Dias, a Professor at University College Dublin, discussing their successful academia (UCD) industry (Aquamarine Power) collaboration. The IT worked beautifully and an engaging and wide ranging discussion was held.

4.4 Publications

The programme was too short for new publications to emerge during the four weeks. However, a wide range of new ideas and interactions emerged and it is expected that these will appear in publications in due course.

There were three events of a introductory and tutorial nature: a shortcourse in Week 1 with talks by Akers, Fokas, and Nicholls; the LMS Spitalfields Day in Week 2 with talks by Groves, Schneider, Shkoller, and Varvaruca; and a Summer School in Week 4 with talks by Ambrose, Bokhove, Bridges, Nachbin, and Wu. Each speaker has agreed to provide lecture notes which will be combined into a book, edited by Bridges, Groves, and Nicholls. A proposal has been submitted to Cambridge University Press for publication in the London Mathematical Society Lecture Notes series. The initial response from CUP has been positive. Final form lecture notes will be submitted for review this autumn.

References

[1] http://newsletter.lms.ac.uk/theory-of-water-waves-a-spitalfields-day-report/

[2] http://sms.cam.ac.uk/collection/1759864

[3] http://waterwaves2014.wordpress.com

[4] http://www-old.newton.ac.uk/reports/casestudies/sww.pdf

[5] http://www.mpe2013.org/

[6] http://www.siam.org/meetings/nw14/