Distributionally Robust Chance-Constrained Generation Expansion Planning

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DTU Electrical Engineering

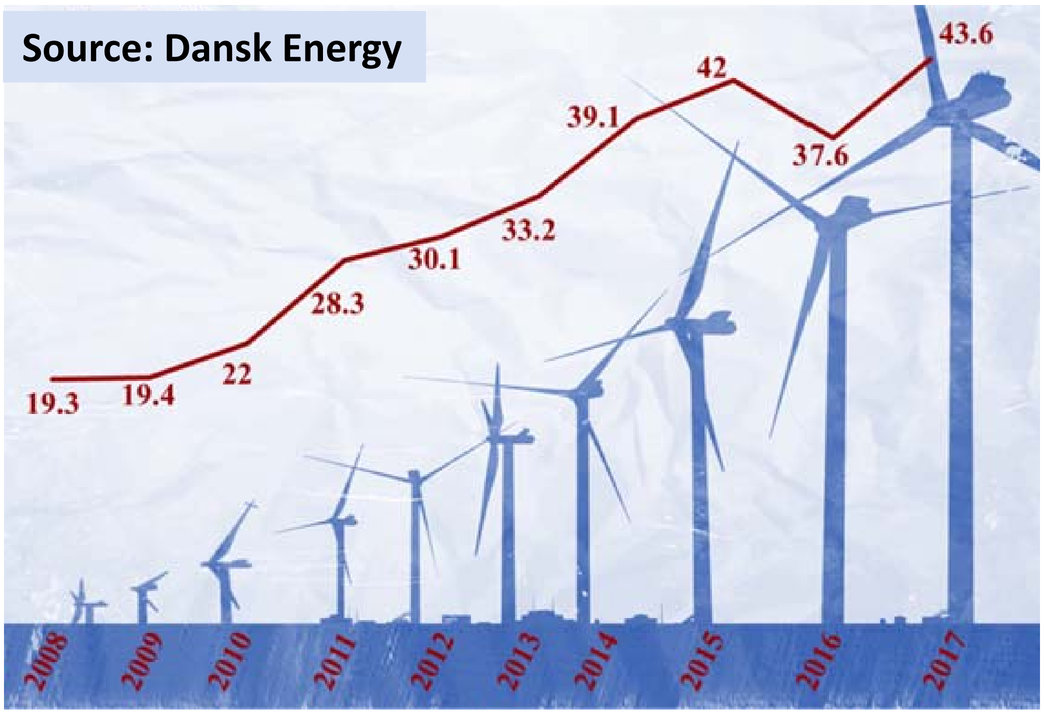
Department of Electrical Engineering

Happy New Year!

Today is the first day of Persian calendar (and the first day of Spring)!

This is happening in Denmark!

Contribution of wind energy to power consumption (%)



In 2017:

- **43.6%** of electricity
- consumption covered by wind
- **1,460 hours** of excess wind

- High uncertainty in supply
- Increased need for operational flexibility in the power systems

The Keywords of This Talk

- **Generation expansion planning problem**

- **Uncertainty**
  - ❑ Long-term uncertainty (e.g., demand growth, regulation policies)
  - ❑ Short-term uncertainty (e.g., wind production, demand)

- **Operational limits**
  - ❑ Unit commitment constraints

Challenges

- The true probability distribution of uncertainty (especially short-term uncertainty) is unknown!

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- By adding more renewables, the net-load profile becomes more and more volatile and uncertain! Can we still ignore the unit commitment constraints in expansion problems?

  - If not, the load duration curve (LDC)-based models may no longer be appropriate [1]-[2].

[1] B. S. Palmintier and M. D. Webster, "Impact of operational flexibility on electricity generation planning with renewable and carbon targets," *IEEE Trans. Sustain. Energy*, vol. 7, no. 2, pp. 672–684, Apr. 2016.

[2] B. Hua, R. Baldick, and J. Wang, "Representing operational flexibility in generation expansion planning through convex relaxation of unit commitment," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 2272–2281, Mar. 2018.

Research Questions

In a generation expansion problem:

How can the **short-term uncertainty** be properly modeled while the true probability distribution is unknown?

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In a generation expansion problem:

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How can the **short-term uncertainty** be properly modeled while the true probability distribution is unknown?

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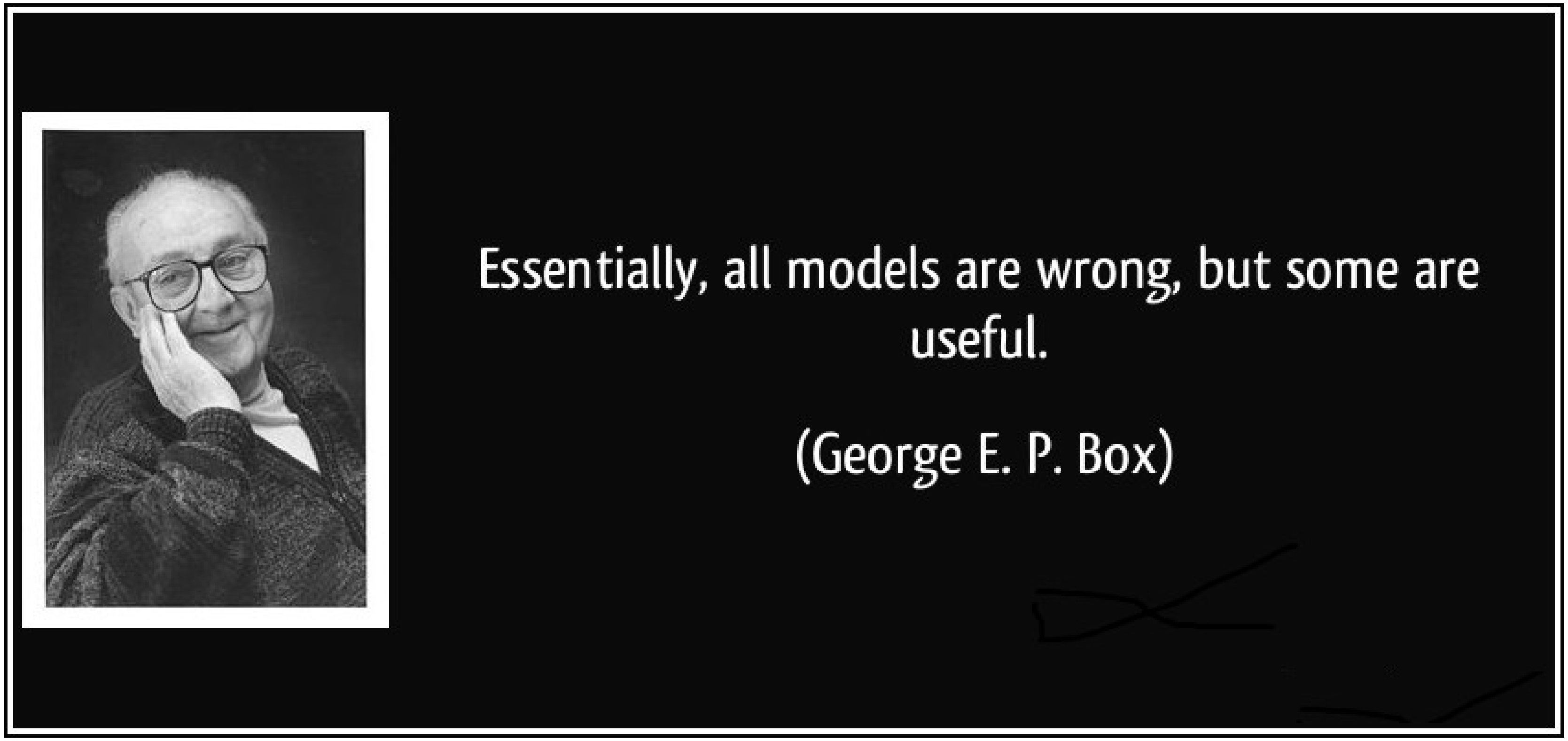
How important is to model the **spatial and temporal correlations** of renewable uncertainty?


Essentially, all models are wrong, but some are useful.
(George E. P. Box)

Outline

- ✓ Background
- ✓ Model
- ✓ Solution Strategy
- ✓ Numerical Study
- ✓ Conclusion and Future Work

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Background

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- **Stochastic programming**
  - Given a finite set of scenarios, it optimizes the problem in expectation.
  - A risk metric (e.g., CVaR) can be incorporated.

Background

Alternatives for modeling uncertainty:

❑ Stochastic programming

- Given a finite set of scenarios, it optimizes the problem in expectation.
- A risk metric (e.g., CVaR) can be incorporated.

To be able to model all potential probability distributions of the short-term uncertainty, a significant (or even infinite) number of scenarios is required!

- Too many scenarios ⟶ Computational issues
- A reduced number of scenarios ⟶ Weak out-of-sample performance

Background

Alternatives for modeling uncertainty:

- **Robust optimization**

  - Given an uncertainty set, it optimizes the problem for the worst-case realization in the set, while keeping the problem feasible for the entire set.

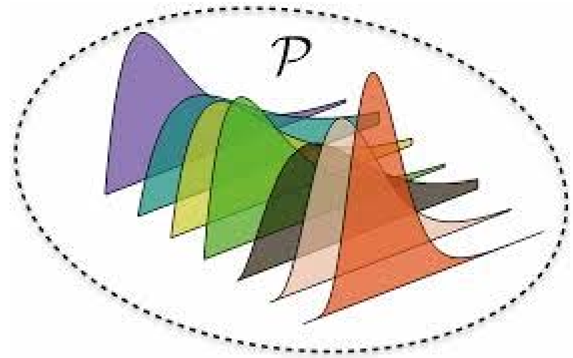
  - Potentially results in a too conservative solution!

Background

Alternatives for modeling uncertainty:

❑ **Distributionally robust optimization**

Given a family of probability distributions (the so-called "ambiguity set") which includes infinite number of distributions,



Source: MIT.edu/vanparys

Background

Alternatives for modeling uncertainty:

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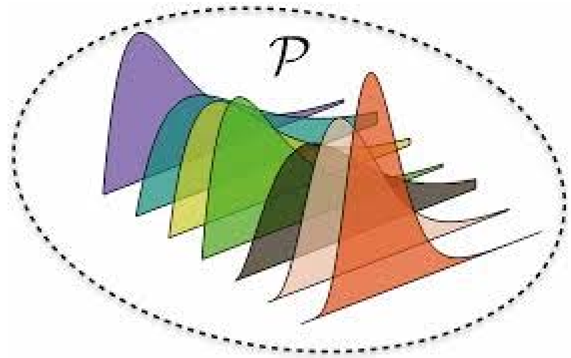

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- it optimizes the problem in expectation for the worst-case probability distribution in the ambiguity set.

- The conservativeness can be adjusted by having chance constraints.

Background

Among others, the two common alternatives in the literature for **building an ambiguity set** are:

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It includes all probability distributions with the same moments (e.g., mean and variance).

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- **Moment-based ambiguity set**

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- **Metric-based ambiguity set**

  It includes all probability distributions whose distance from an empirical distribution is lower than or equal to a given value.

Background

Among others, the two common alternatives in the literature for **building an ambiguity set** are:

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➢ **Metric-based ambiguity set**

It includes all probability distributions whose distance from an empirical distribution is lower than or equal to a given value.

Many interesting talks are available at:
"Distributionally Robust Optimization" Seminar, *Banff International Research Station*, May 2018. https://www.birs.ca/events/2018/5-day-workshops/18w5102/schedule

Outline

- ✓ Background
- ✓ **Model**
- ✓ Solution Strategy
- ✓ Numerical Study
- ✓ Conclusion and Future Work

Model: Time Horizon

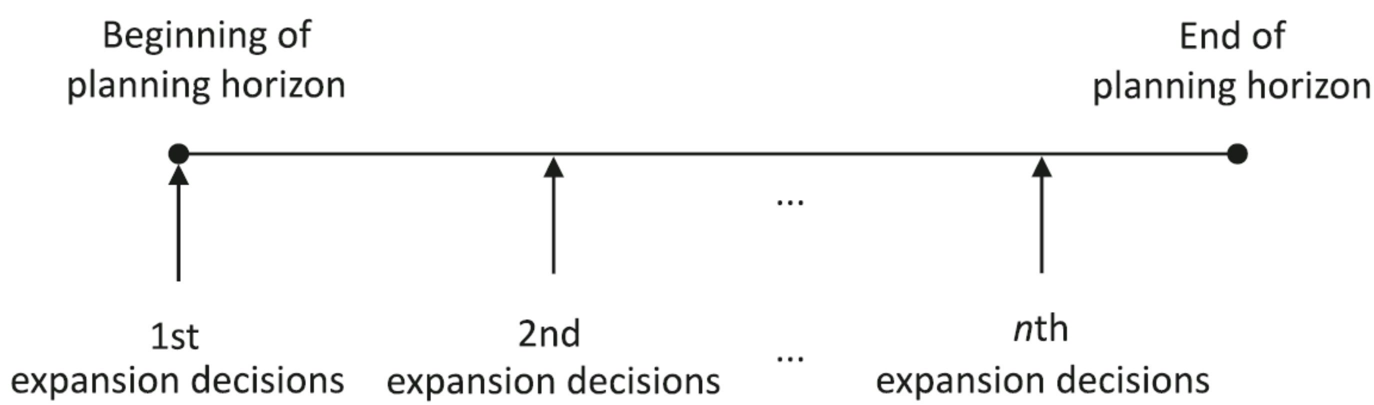
❑ Static (single-stage) model

Source of figures: A. J. Conejo, L. Baringo, JK, and A. S. Siddiqui, “Investment in electricity generation and transmission,” Decision Making Under Uncertainty. Springer, New York, 2016.

Model: Time Horizon

❑ Static (single-stage) model

Remark: A potential extension is a dynamic (multi-stage) model.



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- Static (single-stage) model

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  - Input data: net-load profile over each day
  - Advantage: including unit commitment constraints

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How to cluster and achieve representative days?

[1] K. Poncelet, H. Hschle, E. Delarue, A. Virag, and W. Dhaeseleer, “Selecting representative days for capturing the implications of integrating intermittent renewables in generation expansion planning problems,” *IEEE Trans. Power Syst.*, vol. 32, no. 3, pp. 1936–1948, May 2017.

[2] Y. Liu, R. Sioshansi, and A. J. Conejo, “Hierarchical clustering to find representative operating periods for capacity-expansion modeling,” *IEEE Trans. Power Syst.*, vol. 33, no. 3, pp. 3029–3039, May 2018.

[3] D. A. Tejada-Arango, M. Domeshek, S. Wogrin, and E. Centeno, “Enhanced representative days and system states modeling for energy storage investment analysis,” *IEEE Trans. Power Syst.,* vol. 33, no. 6, pp. 6534–6544, Nov. 2018.

Model: Uncertainty

- Long-term uncertainty:

- Short-term uncertainty:

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- **Long-term uncertainty:**
  - Demand growth uncertainty only,
  - Characterized by *a set of scenarios*
- **Short-term uncertainty:**

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- **Long-term uncertainty:**
  - Demand growth uncertainty only,
  - Characterized by *a set of scenarios*
- **Short-term uncertainty:**
  - Renewable uncertainty only
  - Characterized by *distributionally robust optimization*
  - The renewable forecast for each uncertain source under **long-term scenario *s*, representative day *r*,** and **hour *t*** is

$$\underbrace{\mathbf{m}_{srt}}_{\text{Mean of forecast}} + \underbrace{\gamma_{srt}}_{\text{A random variable for forecast error}}$$

Model: Uncertainty

- **Ambiguity set** under long-term scenario *s*, day *r*, and hour *t* is

$$\mathcal{P}_{srt} = \{\mathbb{P} \in \Psi_{srt}(\mathbb{R}^{|Z|}) : \mathbb{E}(\gamma) = \mu_{srt}, \mathbb{E}(\gamma^\top \gamma) = \Sigma_{srt}\}$$

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Ambiguity set

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Number of uncertain sources

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Ambiguity set

Family of probability distributions

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Ambiguity set

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Mean vector (first moment)

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Mean vector (first moment)

Covariance matrix (second moment)

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Ambiguity set

Family of probability distributions

Mean vector (first moment)

Covariance matrix (second moment)

Remark: We consider the first <u>two</u> moments only (i.e., mean and variance), whose values are <u>exact</u>!

Model

Variables and their dependencies to long- and short-term uncertainties:

Variable	Type	Dependent to the long-term uncertainty?	Dependent to the short-term uncertainty?
Expansion (**y**)	Binary	No	No
Commitment status (**x**)	Binary	Yes	No
Start-up (**u**)	Binary	Yes	No
Production (**p**)	Continuous	Yes	Yes

Note: "No" means there exists a non-anticipativity constraint.

Model

Minimize expected social cost

Subject to

Expansion constraints
(expansion options are discrete)

Operational unit commitment constraints
(including both regular and chance constraints)

Objective Function

$$\min_{\mathbf{y}\in\{0,1\}} \; k^T\mathbf{y} + \sum_s \pi_s \max_{\mathbb{P}\in\mathcal{P}_{srt}} \; \mathbb{E}^{\mathbb{P}}[Q_s(\mathbf{y},\gamma)]$$

Where

$$Q_s(\mathbf{y},\gamma) = \sum_{rt} \min_{\mathbf{p},\mathbf{x},\mathbf{u}} \; w_r(c^T\mathbf{p}_{srt}(\gamma) + h^T\mathbf{u}_{srt})$$

Objective Function

$$\min_{\mathbf{y}\in\{0,1\}} \underbrace{k^T\mathbf{y}}_{\text{Annualized expansion cost}} + \sum_s \pi_s \max_{\mathbb{P}\in\mathcal{P}_{srt}} \mathbb{E}^{\mathbb{P}}[Q_s(\mathbf{y},\gamma)]$$

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Objective Function

Probability of long-term scenario *s*

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(under long-term scenario *s* and under the **worst** short-term probability distribution)

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$$Q_s(\mathbf{y},\gamma) = \sum_{rt} \min_{\mathbf{p},\mathbf{x},\mathbf{u}} w_r(c^T\mathbf{p}_{srt}(\gamma) + h^T\mathbf{u}_{srt})$$

Objective Function

Probability of long-term scenario *s*

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Annualized expansion cost

Expected annual operations cost (under long-term scenario *s* and under the **worst** short-term probability distribution)

Where

Weighting factor of representative day *r*

$$Q_s(\mathbf{y},\gamma) = \sum_{rt} \min_{\mathbf{p},\mathbf{x},\mathbf{u}} w_r(\underbrace{c^T\mathbf{p}_{srt}(\gamma)}_{\text{Production cost}} + \underbrace{h^T\mathbf{u}_{srt}}_{\text{Start-up cost}})$$

Production cost

Start-up cost

Regular Constraints

$$x_{isrt} \leqslant \mathbf{y}_i, \quad \forall i \in G^C, s, r, t$$

Production of candidate units
(if expansion=0, then commitment= 0)

$$-\mathbf{x}_{sr(t-1)} + \mathbf{x}_{srt} - \mathbf{x}_{sr\tau} \leqslant 0,$$
$$\forall \tau \in \{t, ..., MU + t - 1\}, \forall s, r, t$$
$$\mathbf{x}_{sr(t-1)} - \mathbf{x}_{srt} + \mathbf{x}_{sr\tau} \leqslant 1,$$
$$\forall \tau \in \{t, ..., MD + t - 1\}, \forall s, r, t$$

Minimum up- and down-time constraints of units
(both existing and candidate)

$$-\mathbf{x}_{sr(t-1)} + \mathbf{x}_{srt} - \mathbf{u}_{srt} \leqslant 0, \quad \forall s, r, t$$

Start-up constraint of units
(both existing and candidate)

$$\mathbf{1}^\top \mathbf{p}_{srt}(\gamma) + \mathbf{1}^\top(\mathbf{m}_{srt} + \gamma) = \mathbf{1}^\top \mathbf{d}_{srt}, \quad \forall s, r, t$$

Nodal power balance

$$\mathbf{y} \in \{0, 1\}$$
$$\mathbf{x}_{srt}, \mathbf{u}_{srt} \in \{0, 1\}, \quad \forall s, r, t.$$

Expansion, commitment and start-up variables are all binaries!

Chance Constraints

Confidence level

Worst distribution

$$\min_{\mathbb{P}\in\mathcal{P}_{srt}} \mathbb{P}[p_{isrt}(\gamma) \leqslant \overline{p}_i x_{isrt}] \geqslant \underbrace{1-\epsilon_i}_{\text{Confidence level}}, \quad \forall i,s,r,t$$

$$\min_{\mathbb{P}\in\mathcal{P}_{srt}} \mathbb{P}[p_{isrt}(\gamma) \geqslant \underline{p}_i x_{isrt}] \geqslant 1-\epsilon_i, \quad \forall i,s,r,t$$

$$\min_{\mathbb{P}\in\mathcal{P}_{srt}} \mathbb{P}[p_{isrt}(\gamma) - p_{isr(t-1)}(\gamma) \leqslant \overline{r}_i \; x_{isr(t-1)} + rs_i(1-x_{isr(t-1)})] \geqslant 1-\epsilon_i, \quad \forall i,s,r,t$$

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$$\min_{\mathbb{P}\in\mathcal{P}_{srt}} \mathbb{P}[H_l^{\mathrm{G}}\mathbf{p}_{srt}(\gamma) + H_l^{\mathrm{W}}(\mathbf{m}_{srt}+\gamma) - H_l^{\mathrm{D}}\mathbf{d}_{srt} \leqslant \overline{f}_l] \geqslant 1-\epsilon_l, \quad \forall l,s,r,t$$

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Min and max production levels of units (both existing and candidate)

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Ramping limits of units (both existing and candidate)

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Capacity limits of transmission lines

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Capacity limits of transmission lines

Remark: We consider individual (not joint) chance constraints!

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Solution Strategy

For computational simplicity, we relax operational-stage binary variables, but in a tight manner [1]-[2].

[1] B. Hua and R. Baldick, "A convex primal formulation for convex hull pricing," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3814–3823, Sep. 2017.

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$$0 \leqslant \mathbf{x}_{srt} \leqslant 1; \quad 0 \leqslant \mathbf{u}_{srt} \leqslant 1, \quad \forall s, r, t$$

While including **additional** constraints in the problem (in form of chance constraints), tightening the ramping constraints!

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Remark: Expansion decisions are still binary variables!

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Solution Strategy

To reduce the complexity of the problem, the recourse actions are approximated to linear decision rules (affine policy) [1].

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Production of each conventional unit is:

$$\mathbf{p}_{srt} + \alpha_{srt} 1^{\top} \gamma$$

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$$\underbrace{\mathbf{p}_{srt}}_{\text{Tentative schedule (e.g., in day-ahead)}} + \underbrace{\alpha_{srt} 1^\top \gamma}_{\text{Recourse action (as a \textbf{linear} function of forecast error realization)}}$$

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Production of each conventional unit is:

Participation factor (free variable)

$$\underbrace{\mathbf{p}_{srt}}_{\substack{\text{Tentative schedule} \\ \text{(e.g., in day-ahead)}}} + \underbrace{\alpha_{srt} 1^{\top} \gamma}_{\substack{\text{Recourse action (as a }\textbf{linear} \\ \text{function of forecast error} \\ \text{realization)}}}$$

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Reformulation

Objective function can be reformulated in an exact way to a minimization problem [1]:

$$\min_{\mathbf{y},\mathbf{p},\alpha,\mathbf{x},\mathbf{u}} k^T\mathbf{y} + \sum_{s,r,t} \pi_s w_r \{c^T\mathbf{p}_{srt} + h^T\mathbf{u}_{srt}\}$$

[1] E. Delage and Y. Ye, "Distributionally robust optimization under moment uncertainty with application to data-driven problems," *Oper. Res.*, vol. 58, pp. 595–612, 2010.

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Nodal power balance equalities can be reformulated in an exact way to a minimization problem [1]:

$$\mathbf{1}^\top \alpha_{srt} = -\mathbf{1}, \quad \forall s, r, t$$
$$\mathbf{1}^\top \mathbf{p}_{srt} + \mathbf{1}^\top \mathbf{m}_{srt} = \mathbf{1}^\top \mathbf{d}_{srt}, \quad \forall s, r, t$$

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Reformulation

Each chance constraint in a generic form of

$$\min_{\mathbb{P}\in\mathcal{P}} \mathbb{P}\left(v^\top \gamma \leqslant b\right) \geqslant 1-\epsilon$$

can be reformulated in an exact way, resulting in a semi-definite program (SDP) [1].

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Reformulation

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- Under assumptions of having individual chance constraints, and two exact moments only, each individual chance constraints can be reformulated as a second-order cone constraint [1]:

$$v^\top \mu + \sqrt{\frac{1-\epsilon}{\epsilon}} \sqrt{v^\top \Sigma v} \leqslant b$$

[1] E. Delage and Y. Ye, "Distributionally robust optimization under moment uncertainty with application to data-driven problems," *Oper. Res.*, vol. 58, pp. 595–612, 2010.

Reformulation

The resulting model is a

mixed-integer second-order cone problem!

Outline

- ✓ Background
- ✓ Model
- ✓ Solution Strategy
- ✓ **Numerical Study**
- ✓ Conclusion and Future Work

Numerical Study

- **IEEE 118-bus test case:**
  - Existing units: 19 conventional units and 2 wind farms
  - Candidate units: 22 conventional units (four different technologies: nuclear, coal, gas, CCGT)
  - 99 demands, all are inflexible
  - 186 transmission lines
  - Two equiprobable long-term (demand growth) scenarios
  - Under each long-term scenario, the wind penetration (i.e., total wind divided by total load) is 35%

Numerical Study

Input data to model short-term uncertainty

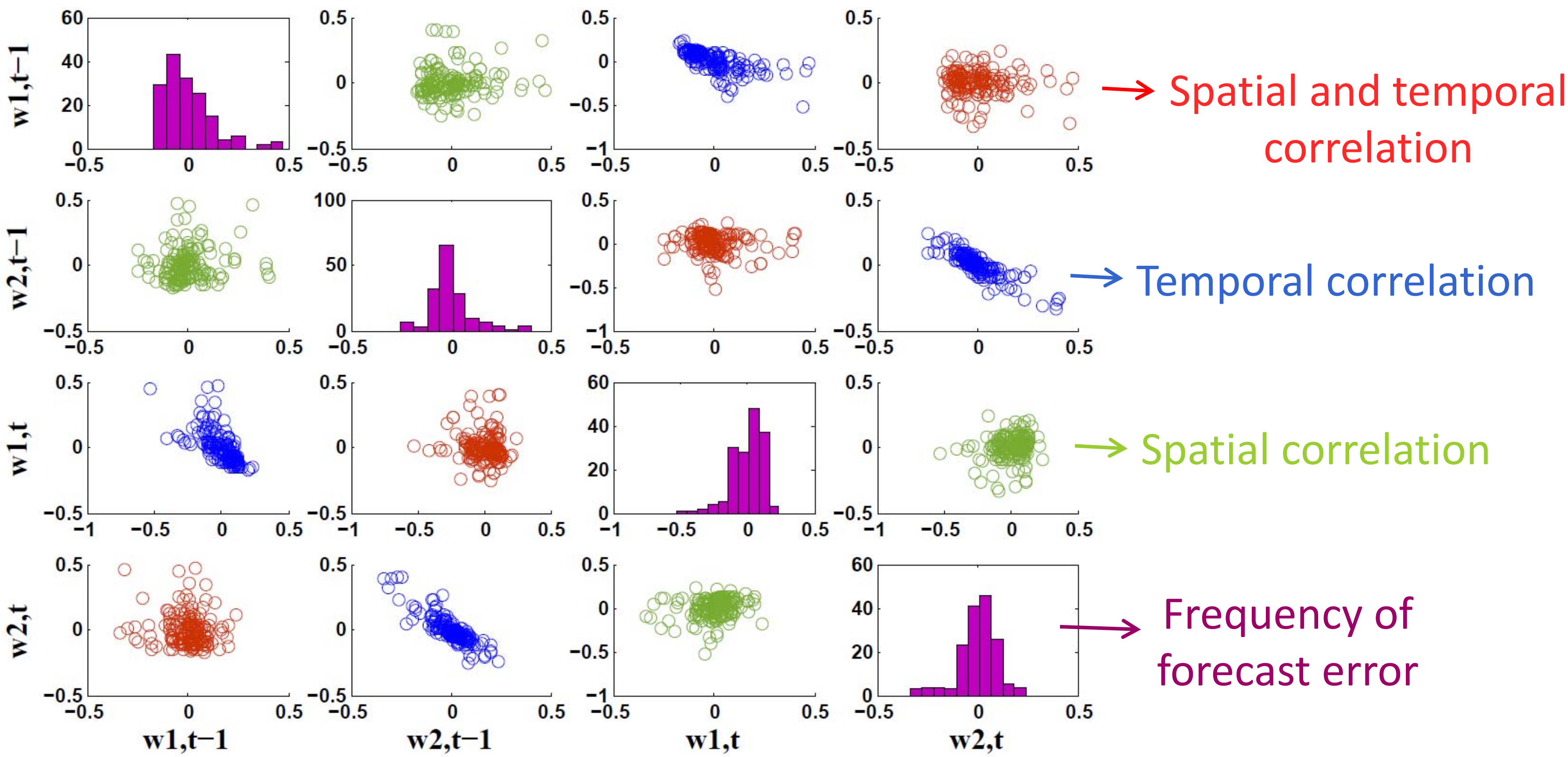
- Hourly wind data (in per-unit) for 10,000 days:

  - Wind data for 5,000 days:
  used for **in-sample study** to train the model, clustered in 10 representative days

  - Wind data for remaining 5,000 days:
  used for **out-of-sample analysis**, again clustered in 10 representative days

Correlation

An example for spatial and temporal correlation of forecast error between 2 wind farms (w1 and w2) and between 2 hours (t-1 and t)



In-sample Results

Remark 1: We assume the same value for ϵ for all chance constraints, and refer to $(1-\epsilon)$ as *confidence level*.

In-sample Results

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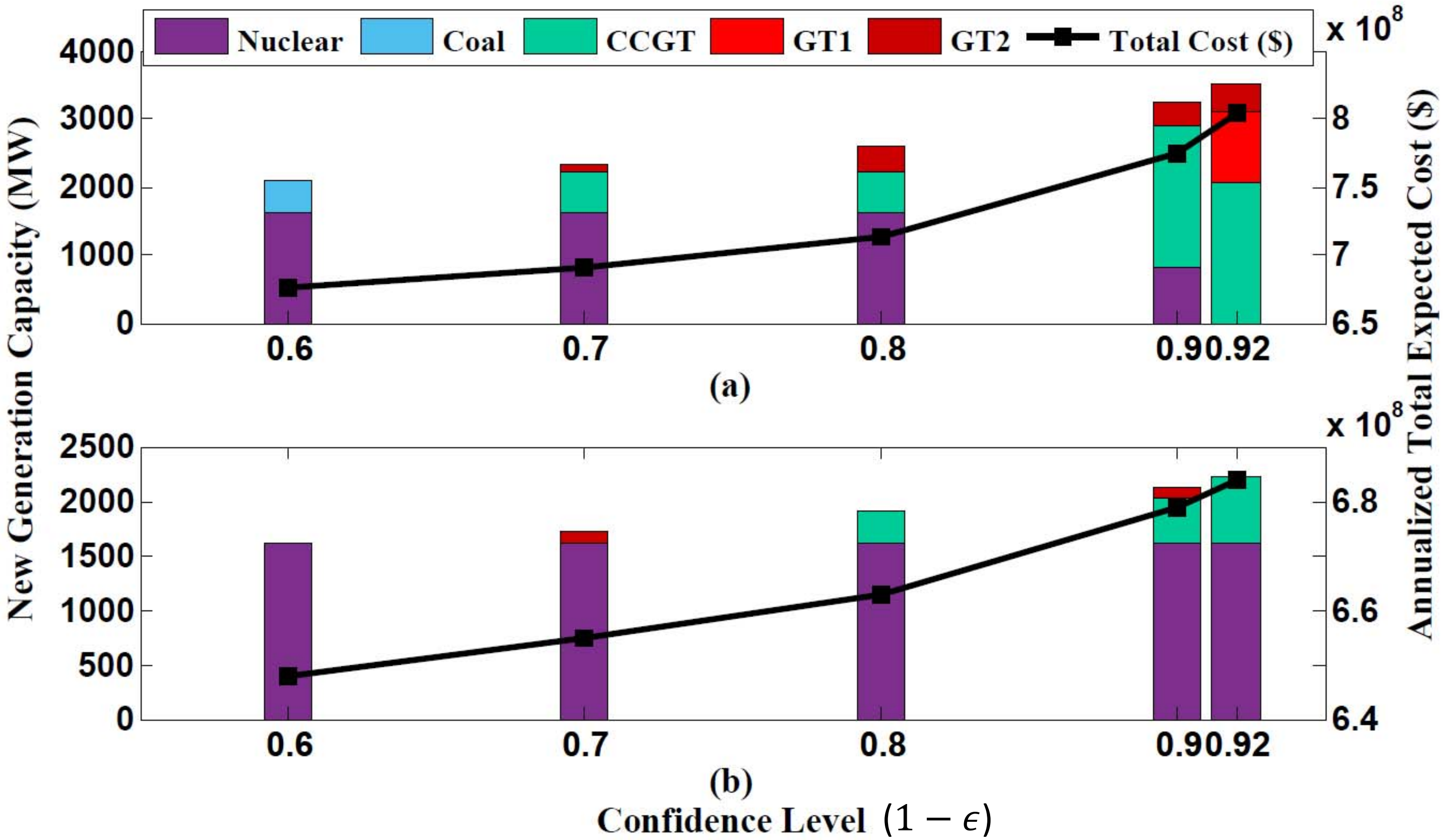
Remark 2: As a benchmark, we consider a chance-constrained model, where the short-term uncertainty follows a normal distribution, with the identical values for the two moments to those in the distributionally robust problem.

This model also results in a mixed-integer second-order cone program [1].

[1] D. Bienstock, M. Chertkov, and S. Harnett, "Chance-constrained optimal power flow: Risk-aware network control under uncertainty," *SIAM Review*, vol. 56, no. 3, pp. 461–495, 2014.

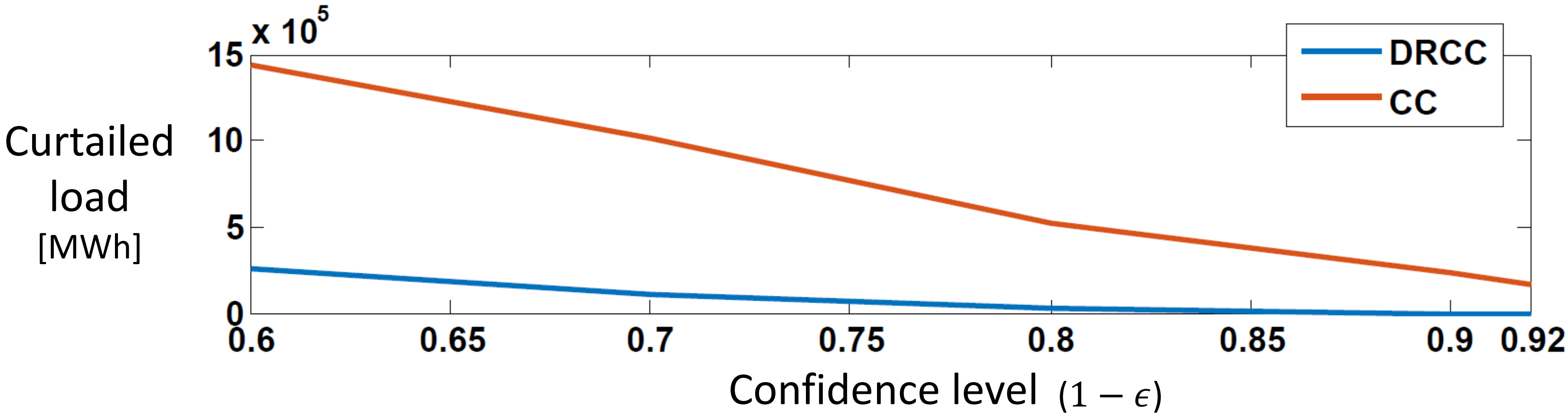
In-sample Results

- **Upper plot:** distributionally robust chance-constrained model
- **Lower plot:** benchmark (chance-constrained model with normal distribution)



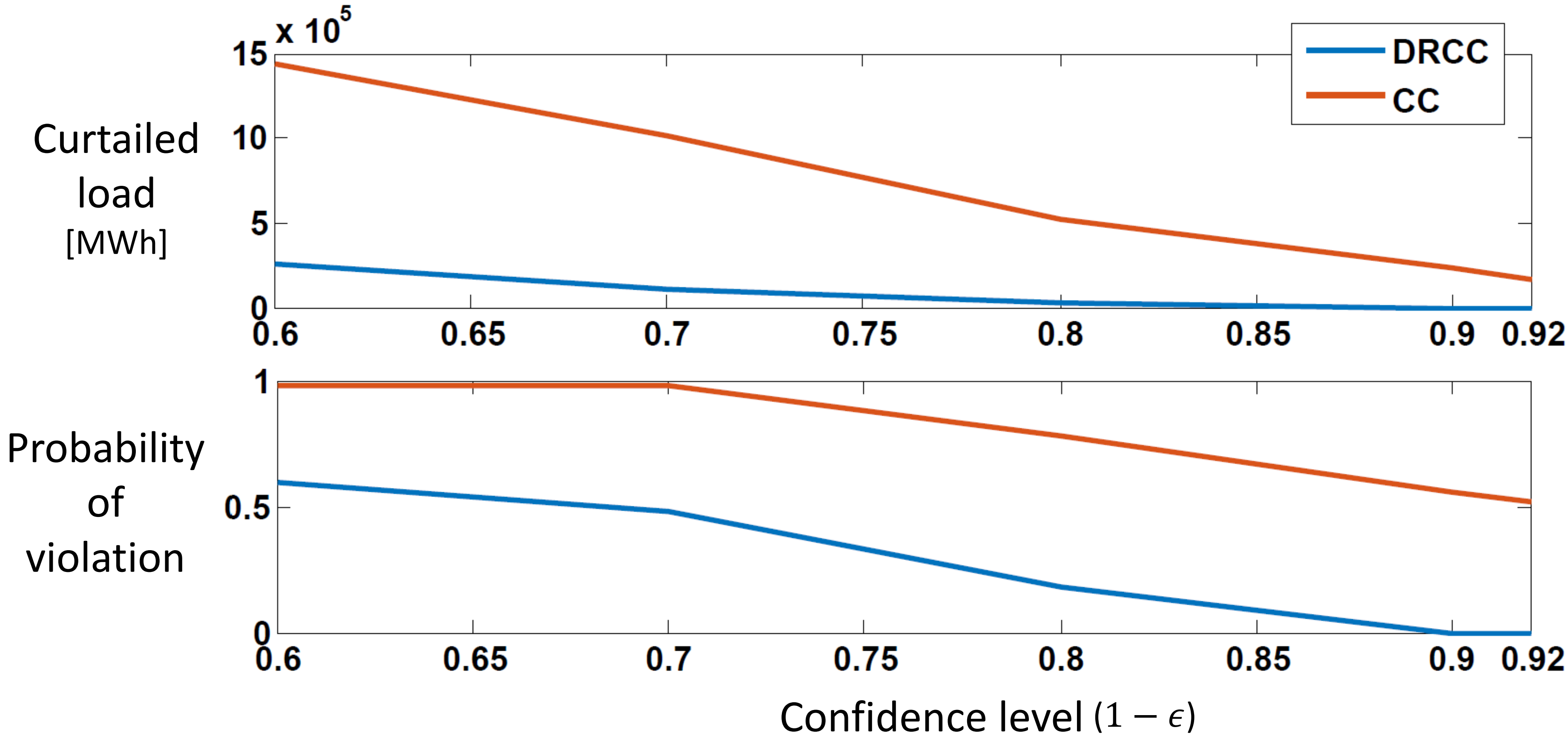
Out-of-sample Results

- **DRCC:** distributionally robust chance-constrained model
- **CC:** benchmark (chance-constrained model with normal distribution)



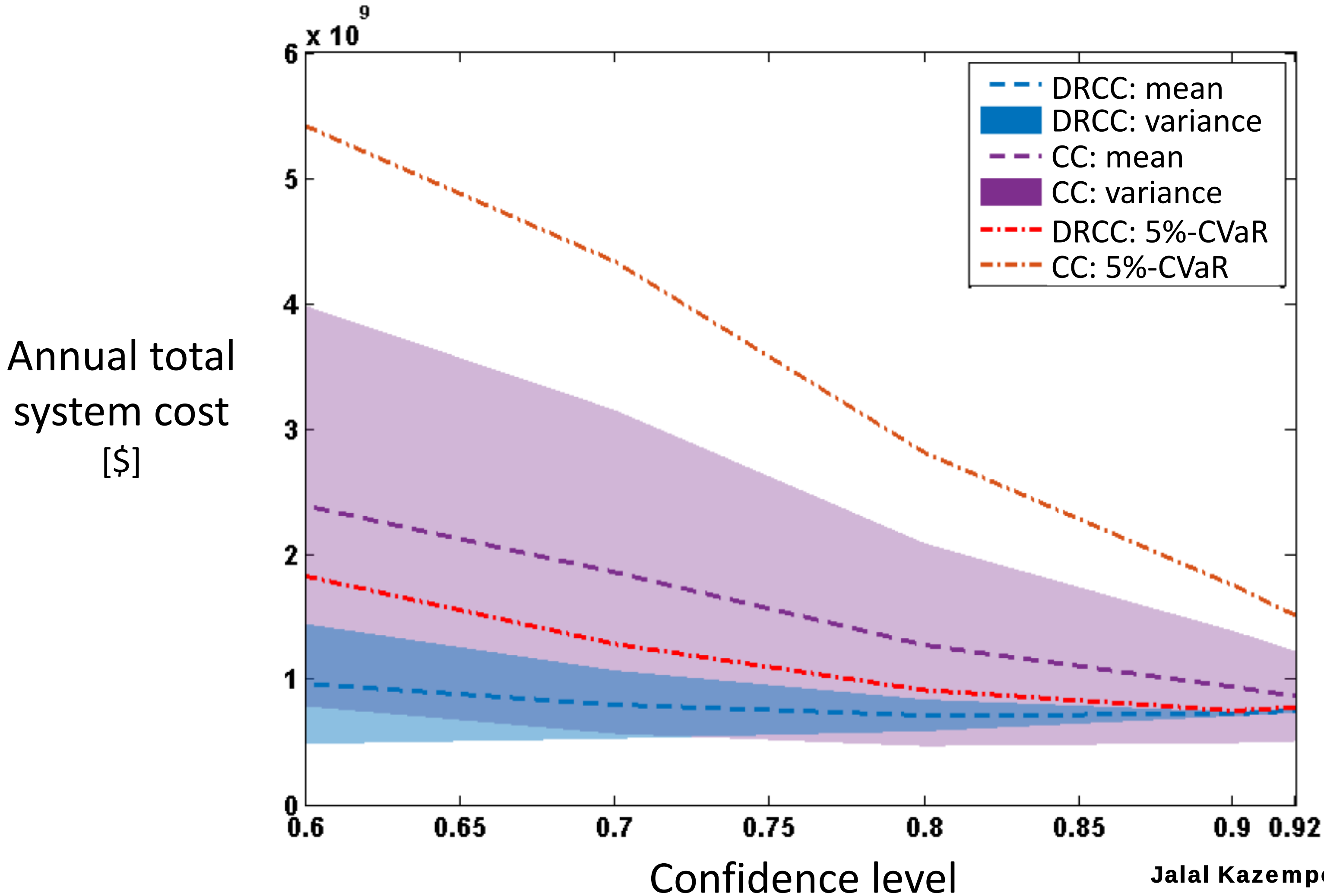
Out-of-sample Results

- **DRCC:** distributionally robust chance-constrained model
- **CC:** benchmark (chance-constrained model with normal distribution)



Out-of-sample Results

- **DRCC:** distributionally robust chance-constrained model
- **CC:** benchmark (chance-constrained model with normal distribution)



Out-of-sample Results

Recall that the long-term uncertainty (demand growth) is modeled by a couple of scenarios!

- Is the current model robust against the long-term uncertainty?

Out-of-sample Results

Recall that the long-term uncertainty (demand growth) is modeled by a couple of scenarios!

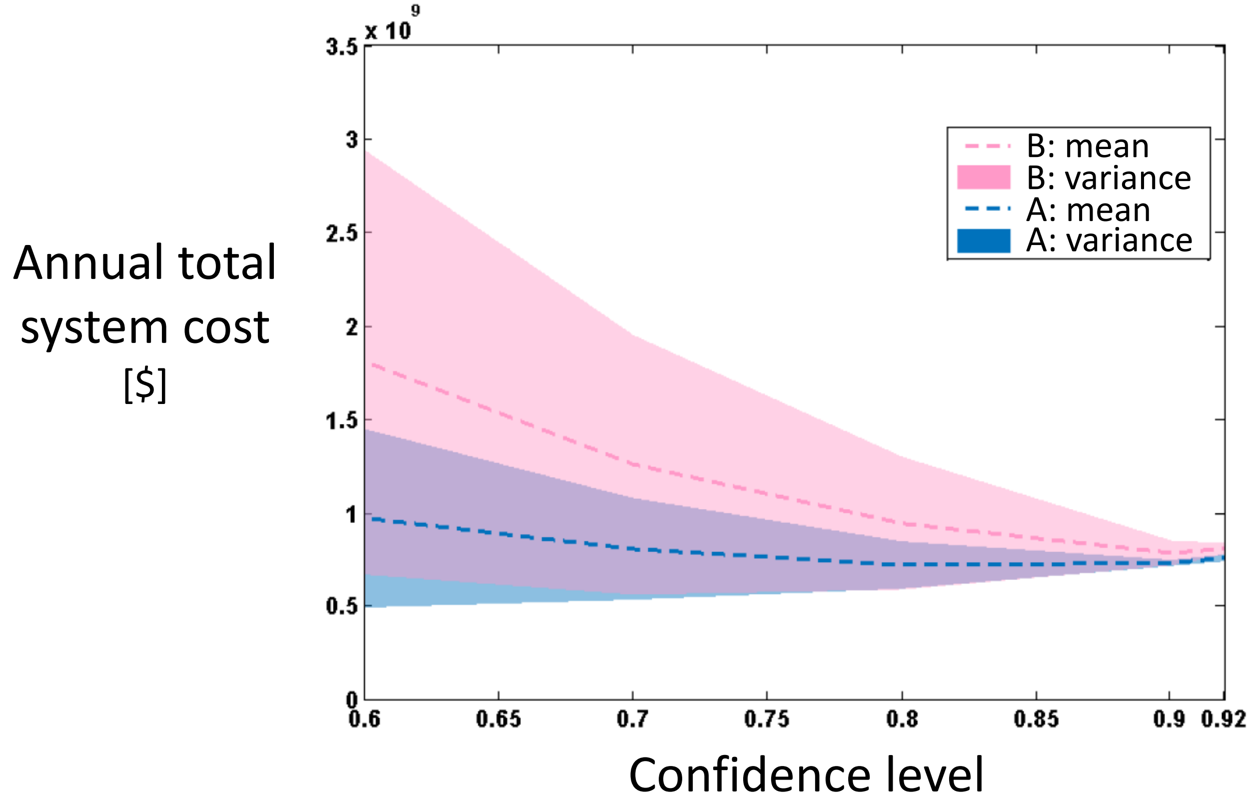
- Is the current model robust against the long-term uncertainty? No!

Out-of-sample Results

Recall that the long-term uncertainty (demand growth) is modeled by a couple of scenarios!

- Is the current model robust against the long-term uncertainty? No!

Case A: the two demand growth realizations are the **same** as the long-term scenarios!
Case B: the two demand growth realizations are **5% higher** than the long-term scenarios!



Computational Issues

- ✓ All simulations are run on an Intel(R) Xeon(R) E5-1650 with 12 processors clocking at 3.50 GHz and 32 GB of RAM.
- ✓ The source code implemented in Matlab using **YALMIP** and solved by **Gurobi 7.5.1**. It will be publicly shared soon!
- ✓ Depending on the confidence level, the computational time is about **90-120 minutes**.

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Conclusion

- If the probability distribution of an uncertain source in generation expansion problem is truly unknown, the distributionally robust optimization is able to provide an appropriate decision-making tool.

- A trade-off between system cost and reliability can be achieved by using chance constraints.

Potential Future Work

- To robustify the model against the long-term uncertainty
- To consider inexact moments and/or joint chance constraints (resulting in an SDP)
- To compare the performance of metric-based vs. moment-based distributionally robust models in an expansion model
- To develop a distributionally robust game-theoretic model for market-based expansion problem (potentially with different values for moments/distance among players)

Thanks for your attention!

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