Double copy in Minitwistor space

Mariana Carrillo González

Machine learning toolkits and integrability techniques in gravity

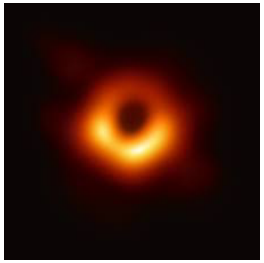
Highly non-linear

Simpler dual descriptions

Gravity

- Complicated scattering

- Hard to construct non-perturbative solutions



- Intersting phenomena!

- Holography
- Double copy

Gravity = (Gauge theory)^2

The BCJ double copy

Bern, Carrasco, Johansson (2008)

$$A^a_\mu : \quad \mathcal{A}_{YM} = \sum_{i\,\epsilon\,\text{trivalent}} \frac{c_i n_i}{d_i}$$

$$h_{\mu\nu},\ \phi,\ B_{\mu\nu} : \quad \mathcal{M}_G = \sum_{i\,\epsilon\,\text{trivalent}} \frac{n_i n_i}{d_i}$$

$$\phi^{a\,a'} : \quad \mathcal{A}_{\phi^3} = \sum_{i\,\epsilon\,\text{trivalent}} \frac{c_i c_i}{d_i}$$

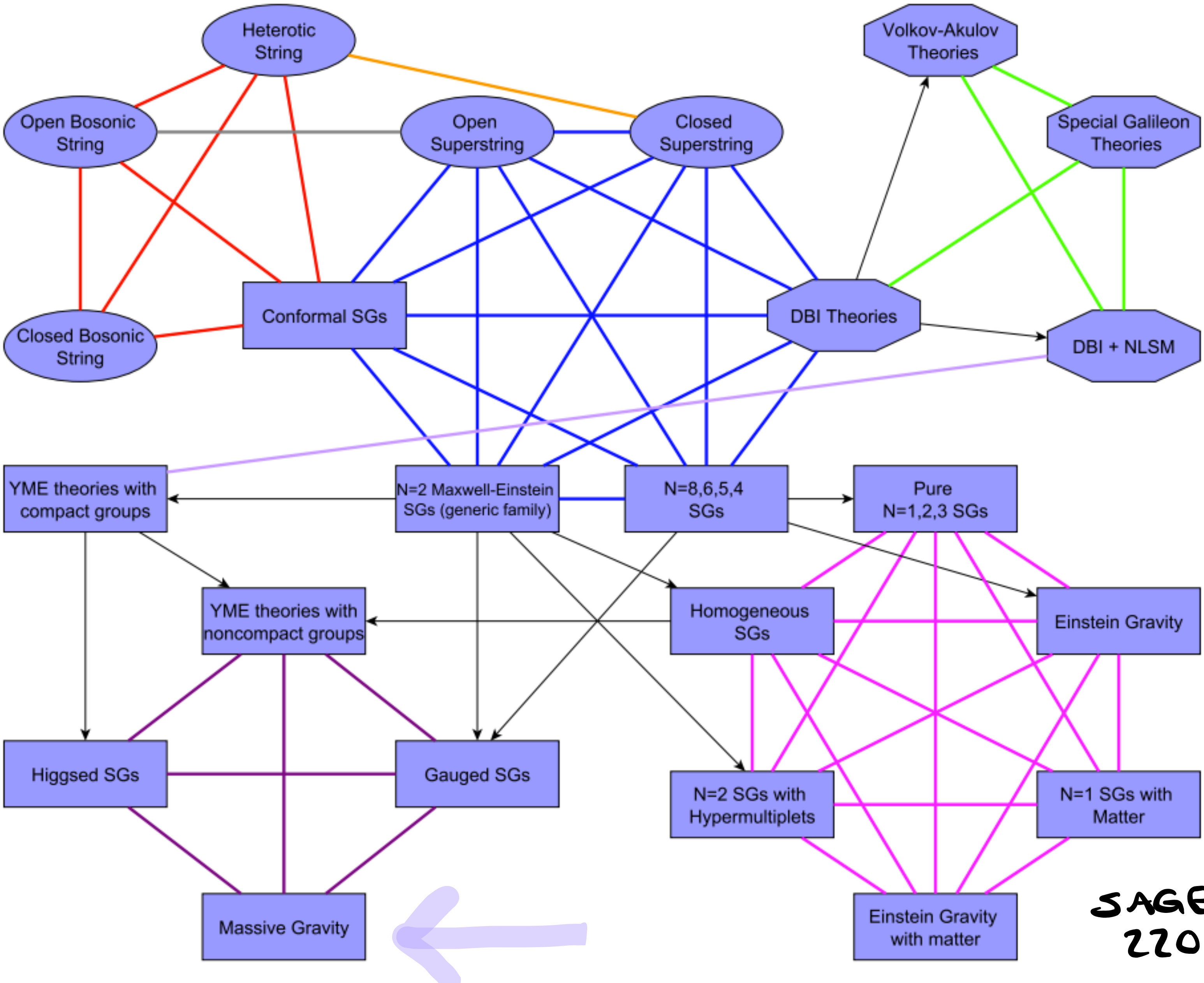
Jacobi relations for color-kinematics duality

$$c_i + c_j + c_k = 0$$

$$n_i + n_j + n_k = 0$$

Amplitudes are invariant under generalized gauge transformations. $n_i \rightarrow n_i + f_i(p^\mu, \epsilon^\mu)$

Web of double copy constructible theories



$= (\mathrm{NLSM})^2$

Simple example

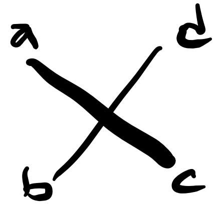
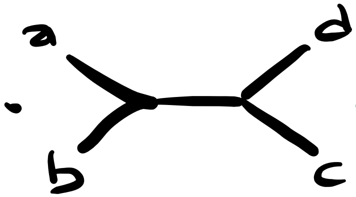
$= (\mathrm{YM})^2$

SAGEX REVIEW
2203.13013

Simple example: longitudinal modes

$$(NLSM)^2 = SGal$$

$$\mathrm{Tr}(T^aT^bT^cT^d)\,\partial_\mu\pi^a\partial^\mu\pi^b\pi^c\pi^d \qquad (\partial\phi)^2(\partial_\mu\partial_\nu\phi)^2$$

contact term  $= s \cdot$  trivalent graph

$$A^{NLSM} = (c_s = f^{abe}f^{ecd}, c_t, c_u)\begin{pmatrix} s & & \\ & t & \\ & & u \end{pmatrix}^{-1}\begin{pmatrix} -s^2 \\ s^2-u^2 \\ u^2 \end{pmatrix} = c^T D^{-1} n$$

$$Mc = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} c_s \\ c_t \\ c_u \end{pmatrix} = 0 \quad \longleftrightarrow \quad Mn = 0$$

CK duality

$$\xrightarrow{\text{D.C.}} A^{SGal} = n^T D^{-1} n = stu$$

BCJ Double Copy

CK dual

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^{\mathsf{T}} D^{-1} n \qquad \mathcal{M}_G = \sum_i \frac{n_i n_i}{s_i + m^2} = n^{\mathsf{T}} D^{-1} n$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-Kinematics duality $\Rightarrow$ $Mn = 0$

For $Mn \neq 0$ use generalized gauge transformations:

$$A^{YM} \rightarrow A^{YM}$$

$n \rightarrow \tilde{n} \equiv n + \Delta n$ s.t. $c^{T} D^{-1} \Delta n = 0$ $\qquad$ $Mn \rightarrow Mn + M D M^{T} v = 0$

$\rightarrow \Delta n = D M^{T} v$

Color-kinematics duality for massive mediators

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BCJ Double Copy

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^T D^{-1} n \quad \xleftrightarrow[\text{dual}]{\text{CK}} \quad \mathcal{M}_G = \sum_i \frac{\tilde{n}_i \tilde{n}_i}{s_i + m^2} = \tilde{n}^T D^{-1} \tilde{n}$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-Kinematics duality $\Rightarrow$ $M\tilde{n} = Mn + MDM^T v = 0$

Invertible

$$\mathcal{M}_G = n^T D^{-1} n + (Mn)^T (MDM^T)^{-1} (Mn)$$

Unphysical poles!

$\Rightarrow$ CK duality for any theory!

Previously observed in:

1701.02519 Bern, Carrasco, Chen, Johansson, Roiban

Generalized D.C.

Massive case:

Johnson-Engelbrecht, Jones, Paranjape

Momeni, Rumbutis, Tolley

Avoiding unphysical poles

$m=0 \xrightarrow{\text{SSB}} m\neq 0$

MDM^T has minimal rank

$\Rightarrow$ 4 Spt BCJ relations

- SSB in Supergravities
  Chiodaroli, Gunaydin, Johansson, Roiban
- Kaluza-Klein theories
  Johnson-Engelbrecht, Jones, Paranjape; Momeni, Rombutis, Tolley; Li, Hang, He
- $(\phi^a)^3$ $U(N)\times G$
  MCG, Liang, Trodden
- Mass-deformed minimal $(DF)^2$
  Johansson, Mogull, Teng; Menezes Carrasco, Pavao

3D Kinematics

$\det(MDM^T) \propto \det(p_i \cdot p_j), \quad i,j<5$

$\Rightarrow \det(MDM^T)^{3d}=0$

$\boxed{\text{Only 1 Spt "BCJ" relation}}$

No spurious poles + correct factorization

Topologically massive theories

MCG, Momeni, Rumbutis
Burger, Emond, Moynihan
Hang, He, Shen

Topologically massive theories

PARITY BROKEN

Topologically Massive Gravity 1 dof

$$S_{TMG} = \frac{1}{\kappa^2}\int d^3x\sqrt{-g}\left(-R - \frac{1}{2m}\varepsilon^{\mu\nu\rho}\left(\Gamma^{\alpha}_{\mu\sigma}\partial_\nu\Gamma^{\sigma}_{\alpha\rho} + \frac{2}{3}\Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\nu\beta}\Gamma^{\beta}_{\rho\alpha}\right)\right)$$

Topologically Massive Yang-Mills 1 dof

$$S_{TMYM} = \int d^3x\left(-\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu} + \varepsilon_{\mu\nu\rho}\frac{m}{12}\left(6A^{a\mu}\partial^\nu A^\rho_a + g\sqrt{2}f_{abc}A^{a\mu}A^{b\nu}A^{c\rho}\right)\right)$$

$1 \otimes 1 = 1$ ✓

Double copy beyond amplitudes?

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In 4d

$$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$$

$$V^\mu = \sigma^\mu_{A\dot{B}} V^{A\dot{B}}$$

$$\sigma^\mu_{A\dot{B}} = (\mathbb{1}, \sigma^i)_{A\dot{B}}$$

Pauli matrices

Weyl tensor

$$W_{\mu\nu\rho\lambda} \to \Psi_{ABCD}\, \epsilon_{\dot{A}\dot{B}}\, \epsilon_{\dot{C}\dot{D}} + \bar{\Psi}_{\dot{A}\dot{B}\dot{C}\dot{D}}\, \epsilon_{AB}\, \epsilon_{CD}$$

self-dual

anti-self-dual

In 3d

$$SO^+(1,2) \cong SL(2,\mathbb{R})/\mathbb{Z}_2$$

$$V^\mu = \sigma^\mu_{3d\,AB} V^{AB}$$

real subset of 4d

Cotton(-York) tensor

$$C_{\mu\nu} = \epsilon_{\mu\rho\sigma} \nabla^\rho \left(R^\sigma_\nu - \tfrac{1}{4}\delta^\sigma_\nu R\right)$$

$$C_{\mu\nu} \longrightarrow C_{ABCD}$$

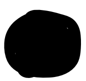
Weyl D.C.	Cotton D.C.
$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$	$SO^+(1,2) \cong SL(2,\mathbb{R})/\mathbb{Z}_2$
$\Psi_{ABCD} = \dfrac{f_{(AB}f_{CD)}}{S}$	$C_{ABCD} = \dfrac{f_{(AB}f_{CD)}}{S}$
Bianchi + eom:	Bianchi + eom:
$\nabla^{A\dot{A}}\Psi_{ABCD} = 0$	$\nabla^{AE} C_{ABCD} = m\, C^{E}{}_{BCD}$
$\nabla^{A\dot{A}} f_{AB} = 0$	$\nabla^{AE} f_{AB} = m\, f^{E}{}_{B}$
$(\Box - R/6) S = 0$	$(\Box - R/6 - m^2) S = 0$

Dimensional reduction?

Weyl double copy from amplitudes double copy

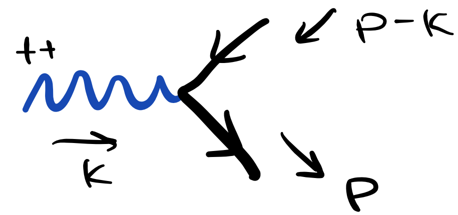
In $(--++)$ signature: $\quad k^\mu \to k_A \bar{k}_{\dot{B}}$

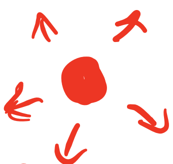
Monteiro, O'Connell, Peinador Veiga, Sergola



Black Hole

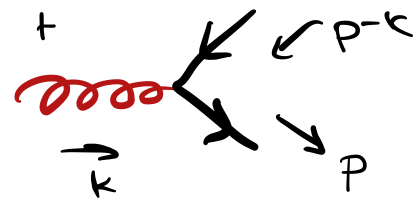
$$\Psi_{ABCD} = \mathrm{Re}\int_k k_A k_B k_C k_D \, \mathcal{M}_3^{++}(k)\, e^{-ik\cdot x}$$



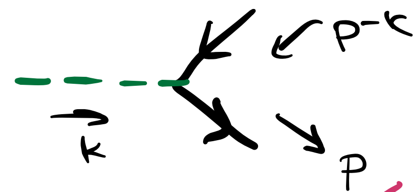


Electric charge

$$f_{AB} = \mathrm{Re}\int_k k_A k_B \, A_3^{+}(k)\, e^{-ik\cdot x}$$



$$S = \mathrm{Re}\int_k A_3^{\phi}(k)\, e^{-ik\cdot x}$$



If convolution turns into product!

KLT DC $\quad \mathcal{M}_3^{++} = \dfrac{A_3^{+} A_3^{+}}{A_3^{\phi}} \Rightarrow \Psi_{ABCD} = \dfrac{f_{(AB} f_{CD)}}{S}$ Weyl DC

Luna, Moynihan, White

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In $(- - +)$ signature:

2212.04783 MCG, Emond, Moynihan, Rombotis, White

KLT DC

$$\mathcal{M}_3^{++} = \frac{A_3^+ A_3^+}{A_3^\phi} \Rightarrow C_{ABCD} = \frac{f_{(AB} f_{CD)}}{S}$$

Cotton DC

linearized anyons sol X

Waves ✓

$$\Psi_4 = \frac{m}{2} \frac{\Phi_2^2}{\phi}$$

$$C_{ABCD} = \Psi_4 o_A o_B o_C o_D$$

$$f_{AB} = \Phi_2 o_A o_B$$

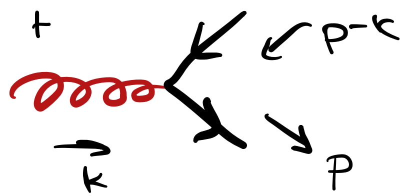
2202.10476 MCG, Momeni, Rombotis — Cotton DC momentum space ✓

$H^1(\mathbb{PT},\mathcal{O}(n\ 1))$

twistor space

Half Fourier transform

special representatives



Penrose transform

inverse Fourier transform

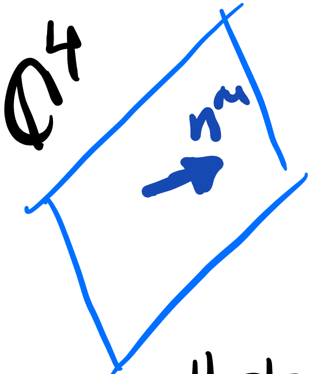
position space

Special cases

$F_{\mu\nu}(x)$

Twistor space

$\mathbb{C}^4$



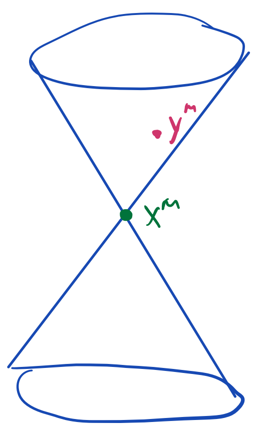
null planes

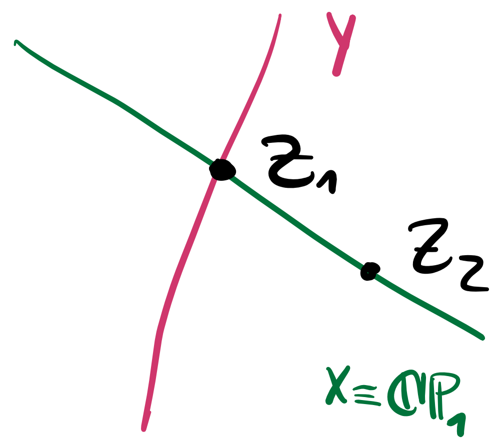
$$u = n_\mu x^\mu = \lambda_A \bar{\lambda}_{\dot{A}} x^{A\dot{A}} \rightarrow \boxed{\mu^{\dot{A}} = \lambda_A x^{A\dot{A}}}$$

incidence relation

$$Z^\alpha = (\mu^{\dot{A}}, \lambda_A) \in \mathbb{CP}_3$$

$$Z \sim r Z, \quad r \in \mathbb{C}^*$$





$$X^{\alpha\beta} = Z_1^{[\alpha} Z_2^{\beta]} \leftarrow$$ info. on x^μ up to scalings

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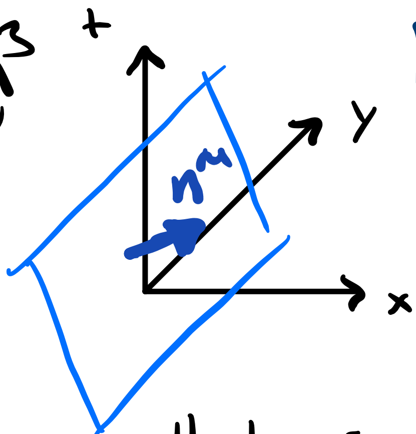
$\mathbb{R}^3$



null planes

$\Downarrow$

$u = n_\mu x^\mu, \quad n_{AB} = n_\mu \sigma^\mu_{AB} = \lambda_A \lambda_B$

Minitwistor space

$$\mathbb{MT} = \{ Z^\alpha = (u, \lambda_A), \; u = x^{AB}\lambda_A\lambda_B \}$$

$$\cong \frac{ESL(2,\mathbb{C})}{Q}$$

isometries null planes

$$(u, \lambda_A) \sim (r^2 u, r\lambda_A)$$

From Dim. Red. $x^{4d}_{A\dot{A}} S_B{}^{\dot{A}} = x^{3d}_{AB} + iz\,\epsilon_{AB}$

$$\mu^{\dot{A}} = \lambda_A x^{A\dot{A}}$$

$$(x^{AB}\lambda_A\lambda_B, \lambda_A) \sim (r^2 (x^{AB}\lambda_A + iz\lambda^B)\lambda_B, r\lambda_A)$$

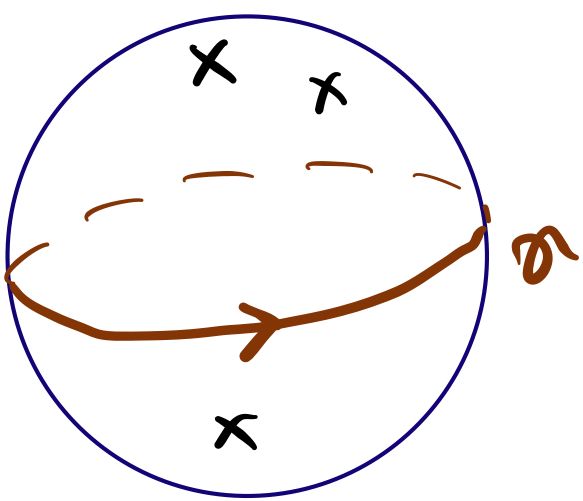
Penrose Transform

spin n field

$$\underbrace{\phi_{AB\cdots D}}_{2n} = \frac{1}{2\pi i}\oint_{\Gamma} \langle \lambda d\lambda \rangle \, \lambda_A \cdots \lambda_D \, f(Z)\Big|_x$$

where $f(r^2 u, r\lambda_A) = r^{-2n-2} f(x^{AB}\lambda_A\lambda_B, \lambda_A)$

$$\in H(\mathbb{MT}, \mathcal{O}(-2n-2))$$



$$\boxed{\nabla^{H}{}_{A}\,\phi_{HB\cdots D} = 0}$$

spin n field

$$\underbrace{\phi_{AB\cdots D}}_{2n} = \frac{1}{2\pi i}\oint_{\Gamma} \langle\lambda d\lambda\rangle\, \lambda_A\cdots\lambda_D\, f(Z)\Big|_X$$

complex rep. of Q

$$\text{where } f\left(r^2\left(X^{AB}\lambda_A + iz\lambda^B\right)\lambda_B,\ r\lambda_A\right) = r^{-2n-2}\, e^{-izm}\, f\left(X^{AB}\lambda_A\lambda_B,\ \lambda_A\right)$$

$$\Rightarrow f(Z) = e^{-m\frac{\langle a|X|\lambda\rangle}{\langle a\lambda\rangle}}\, g(u,\lambda_\alpha) \quad \in {}^{1}H\left(\mathbb{MT}, \mathcal{O}(-2n-2,m)\right)$$

$$\boxed{\nabla^H{}_A\, \phi_{HB\cdots D} = m\, \phi_{AB\cdots D}}$$

Choose twistor representatives for type N (waves)

$$\boxed{f_{-6} = f_{-4}^{2} / f_{-2}}$$

DOUBLE COPY

$$f_{-2-2n} = \frac{e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}}{\chi(u,\lambda_\alpha)\left(\xi(u,\lambda_\alpha)\right)^{2n+1}}\, g(u,\lambda_\alpha)$$

$$\lambda_\alpha = (1, z)$$

$$\phi_{\underbrace{AB\cdots D}_{2n}} = \frac{1}{2\pi i}\oint dz\, \lambda_A \cdots \lambda_D \frac{e^{-m\, q(x;z)}}{(z-z_0)(z-z_1)^{2n+1}}\, g(x;z)$$

$$= \lambda^0_A \cdots \lambda^0_D \frac{g(x;z_0)}{(z_0-z_1)^{2n+1}} e^{-m\, q(x;z_0)}$$

$\Rightarrow$ Cotton DC

Choose twistor representatives for type D (field around isolated objects)

$$\boxed{f_{-6} = f_{-4}^{2} / f_{-2}}$$

DOUBLE COPY

$$f_{-2-2n} = \frac{e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}}{\left(\chi(u,\lambda_\alpha)\,\xi(u,\lambda_\alpha)\right)^{1+n}} \qquad \lambda_\alpha = (1,z)$$

$$\phi_{\underbrace{AB\cdots D}_{2n}} = \frac{1}{2\pi i}\oint dz\,\lambda_A\cdots\lambda_D\,\frac{e^{-mq(x;z)}}{(z-z_0)^{1+n}(z-z_1)^{1+n}}$$

$$\propto \lim_{z\to z_0}\frac{d^{2n}}{dz^{2n}}\left(\frac{z^r e^{-mq(x;z)}}{(z-z_1)^{1+n}}\right)$$

! No squaring relation in position space

Derivation from twistor space

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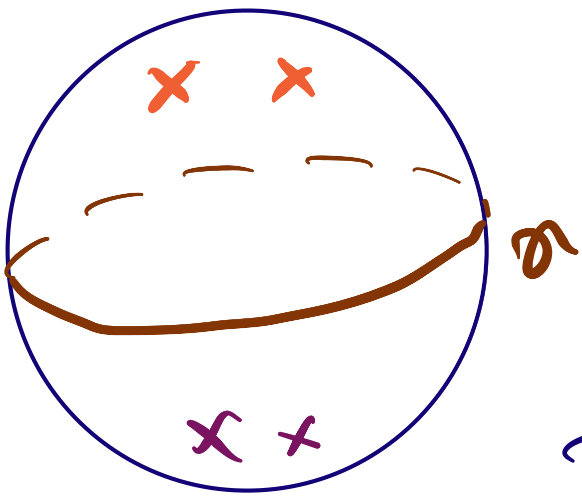
DOUBLE COPY

$$f_{-6} = f_{-4}^{2} / f_{-2}$$



$f \leftarrow$ cohomology representative

$$f \to f + g_N - g_S$$

leaves $\phi_{1\ldots D}$ invariant but breaks double copy relation.

Representatives are related to amplitudes

Cohomology representatives from amplitudes

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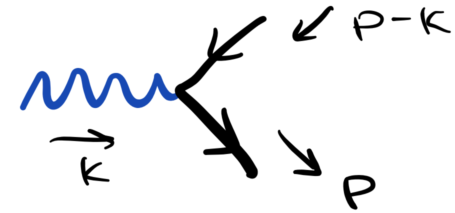
$$k^\mu \to \omega\, \lambda_{(A}\bar{\lambda}_{B)} \qquad \epsilon^\mu \to \frac{\omega}{m}\lambda_A \lambda_B$$



$$d^3k = 2\omega^2 d\omega \langle\lambda\bar{\lambda}\rangle \langle\lambda d\lambda\rangle \langle\bar{\lambda} d\bar{\lambda}\rangle$$

$$\delta(k^2 - m^2)\,\Theta(\omega) \to \delta\left(\omega - \frac{m}{\langle\lambda\bar{\lambda}\rangle}\right)$$

Perform half F.T.

$$\psi_{ABCD} = \oint \langle\lambda d\lambda\rangle \lambda_A \lambda_B \lambda_C \lambda_D\, \omega^3 \left(M_3^{++}/\omega^2\right) e^{-ik\cdot x}\, \delta\left(\omega - \frac{m}{\langle\lambda\bar{\lambda}\rangle}\right) d\omega$$

$$f_{AB} = \oint \langle\lambda d\lambda\rangle \lambda_A \lambda_B\, \omega^2 \left(A_3^{+}/\omega\right) e^{-ik\cdot x}\, \delta\left(\omega - \frac{m}{\langle\lambda\bar{\lambda}\rangle}\right) d\omega$$

$$S = \oint \langle\lambda d\lambda\rangle\, \omega\, A_3^{\phi}(k)\, e^{-ik\cdot x}\, \delta\left(\omega - \frac{m}{\langle\lambda\bar{\lambda}\rangle}\right) d\omega$$

Cohomology representatives from amplitudes

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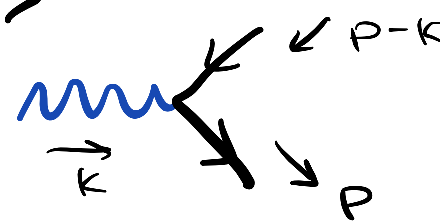
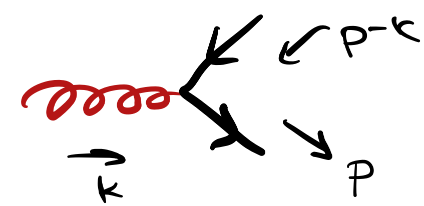
Anyons $\longrightarrow$ Type D at linear order

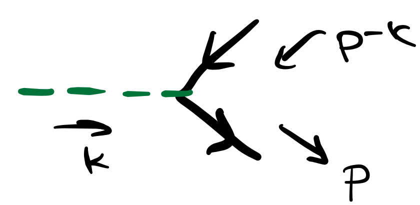
$$2\Psi_{ABCD} \propto \oint \langle \lambda d\lambda \rangle \lambda_A \lambda_B \lambda_C \lambda_D \frac{1}{\langle a\lambda\rangle^3} e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}$$



$$f_{AB} \propto \oint \langle \lambda d\lambda \rangle \lambda_A \lambda_B \frac{1}{\langle a\lambda\rangle^2} e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}$$



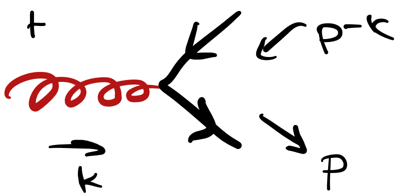
$$S \propto \oint \langle \lambda d\lambda \rangle \frac{1}{\langle a\lambda\rangle} e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}$$



Double copy in different spaces

momentum space ✓

Using CK duality



Half Fourier transform

twistor space

$H^1(\mathbb{PT}, \mathcal{O}(-4))$

special representatives

Penrose transform

inverse Fourier transform

position space

special cases

$F_{\mu\nu}(x)$

$$(\mathrm{TMYM})^2 = \mathrm{TMG}$$

Robust evidence: Momentum, Coordinate, Twistor space

Open Questions:

- Type D? (Warped AdS)
- Origin via dimensional reduction?

→ Minitwistor space of AdS_3

Conclusions and future directions

Non-trivial backgrounds

scalar EFT's

most general EFTs?

Classical solutions

KK modes

SUSY

strings

Double Copy

$$A^{\mu a} \otimes A^{\mu a} = h_{\mu\nu} + B_{\mu\nu} + \phi$$

Other massive theorres?

¿ D=2

D=3

YME, Higgsed sugra, Conformal sugra, ...

Topologically massive theories