An update on holography of information

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Collaborators and References

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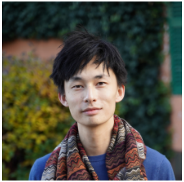

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- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

Collaborators and References

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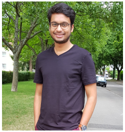

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- `2002.02448`, Alok Laddha, Siddharth Prabhu, Pushkal Shrivastava, S.R.
- `2008.01740`, Chandramouli Chowdhury, Olga Papadoulaki, S.R.
- `2107.14802`, Chandramouli Chowdhury, Victor Godet, Olga Papadoulaki, S.R.

Collaborators and References

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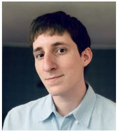

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- `2107.03390`, Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi
- `2012.04671`, Geng, Karch, Perez-Pardavila, S.R., Randall, Riojas, Shashi

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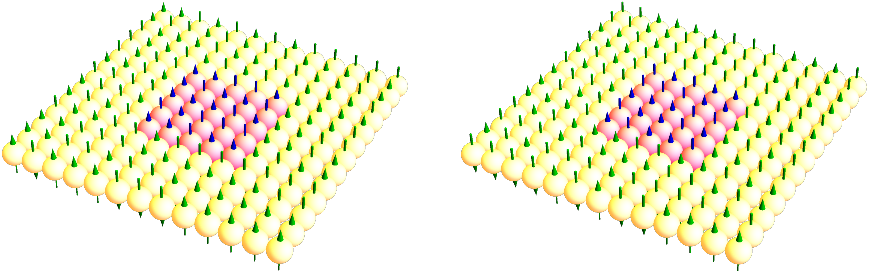
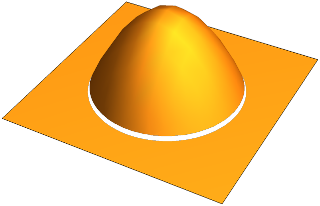
- `2012.05770`, S.R.
- `2110.05470`, S.R.

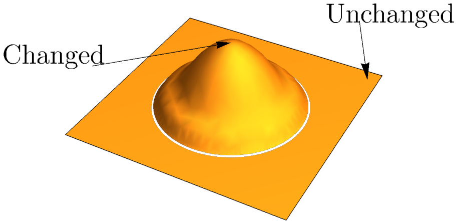
The split property in nongravitational theories



- In a nongravitational theory, the state inside and outside a bounded region can be specified independently. (Clear in a lattice regularization.)

Ordinary localization of information in QFT

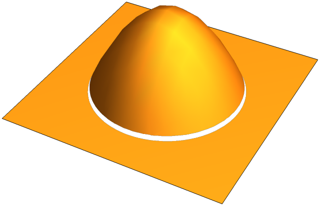


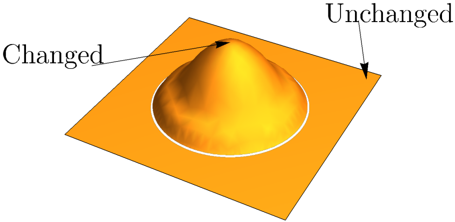


The split property is key to our usual idea that information is localized “inside” some region and is not available in its complement.

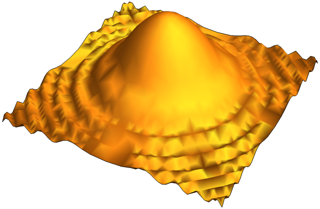
Complement-of-bounded-region has info about region

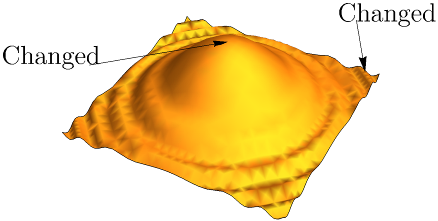
Nongravitational QFT:





Gravity:

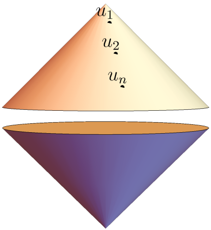


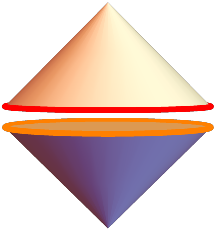


Overview

- This unusual localization of information can be inferred from a gravitational analysis.
- For simple states, this effect is visible perturbatively.
- This effect is important for the black-hole information problem.
- In some regimes, this effect becomes unimportant and gravity behaves like an ordinary QFT.
- Our results pertain to information, and not a full holographic dual. So we term this “holography of information.”

Holography of information in asymptotically flat space





In a nongravitational theory, asymptotic algebra at $\mathcal{I}^+$ comprises independent operators

$$\text{span}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}$$

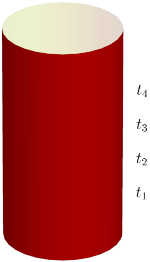
In gravity, any operator on $\mathcal{I}^+$ can be approximated arbitrarily well from

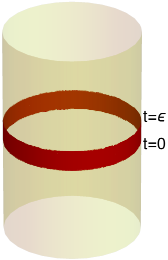
$$\text{span}\{\mathcal{Y}(u_1)\ldots\mathcal{Y}(u_n)\},$$

$$u_i \in [-\infty, -\frac{1}{\epsilon}]$$

Laddha, Prabhu, S.R., Shrivastava (2020); Marolf (2006–13)

Holography of information in asymptotically AdS





In a nongravitational theory, asymptotic algebra lives on the entire boundary

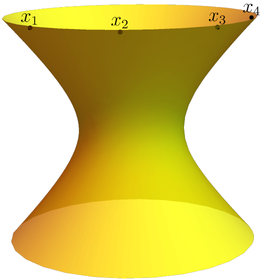
$$\text{span}\{\mathcal{Y}(t_1)\mathcal{Y}(t_2)\ldots\mathcal{Y}(t_n)\}$$

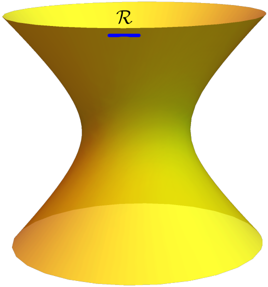
In gravity, any asymptotic operator can be approximated arbitrarily well from

$$\text{span}\{\mathcal{Y}(t_1)\ldots\mathcal{Y}(t_n)\},$$
$$t_i \in [0, \epsilon]$$

Laddha, Prabhu, S.R., Shrivastava (2020); R.de Mello Koch, Kemp (2022)

Holography of information in asymptotically dS





In a nongravitational theory, complete set of observables are

$$\langle \mathcal{Y}(x_1) \ldots \mathcal{Y}(x_n) \rangle$$

In gravity, complete set of observables are

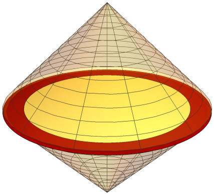
$$\langle \mathcal{Y}(x_1) \ldots \mathcal{Y}(x_n) \rangle, x_i \in \mathcal{R}$$

[Chakraborty, Chakravarty, Godet, Paul, S.R., 2023]

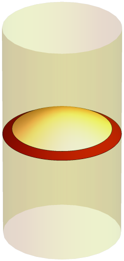
Physical interpretation: picture

Precise results are in terms of asymptotic algebras.

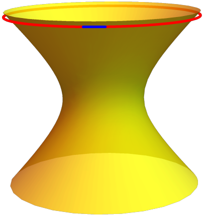
But physically, they imply that information about the whole state is available in a subregion.



Flat space



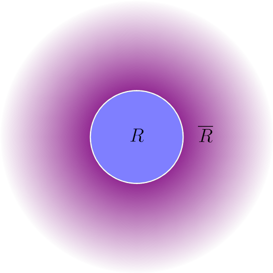
AdS



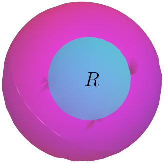
dS

Physical interpretation: picture

AdS and flat space



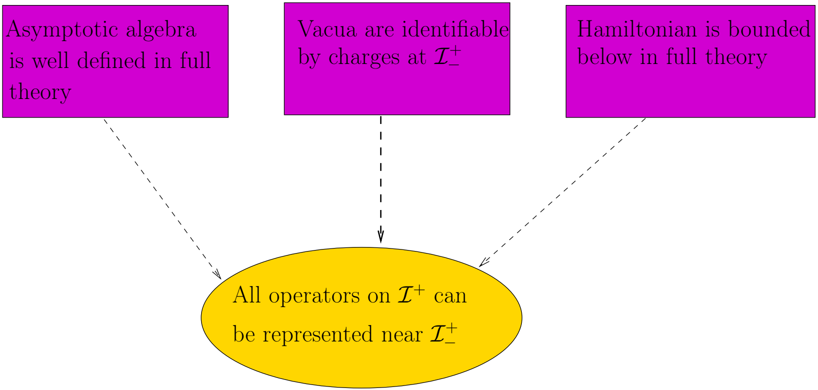
dS



The complement of a bounded region has all information about the state.

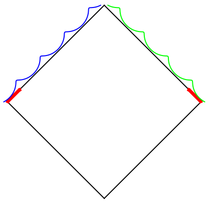
Assumptions: flat space

These results rely on weak assumptions about the full theory.



Elements of proof: algebra

Assumption: Asymptotic algebra makes sense in the UV theory.



- Study Hilbert space obtained by asymptotic quantization

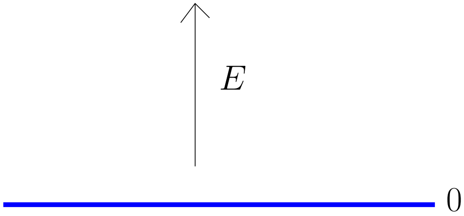
$$\mathcal{H}^{(0)} = \text{span of}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}|0\rangle, \quad u_i \in (-\infty, \infty)$$

- Define

$$\mathcal{A}_{-\infty,\epsilon} = \text{span of}\{\mathcal{Y}(u_1)\mathcal{Y}(u_2)\ldots\mathcal{Y}(u_n)\}, \qquad u_i \in (-\infty, -\frac{1}{\epsilon}]$$

Elements of proof: entanglement

Assumption: Hamiltonian is bounded below.



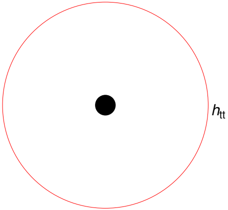
$\Longrightarrow$ any state $|n\rangle \in \mathcal{H}^{(0)}$ is approximated arbitrarily well by

$$|n\rangle \doteq X_n|0\rangle$$

where $X_n \in \mathcal{A}_{-\infty,\epsilon}$.

Elements of proof: Projector onto the vacuum

Assumption: Vacuum can be identified by a boundary operator.



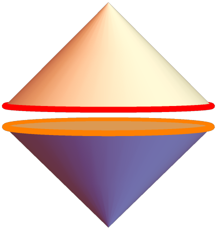
$$P_0 = |0\rangle\langle 0| \in \mathcal{A}_{-\infty,\epsilon}$$

A unique property of gravity is that the vacuum can be selected from the asymptotic region. Follows from gravitational constraints.

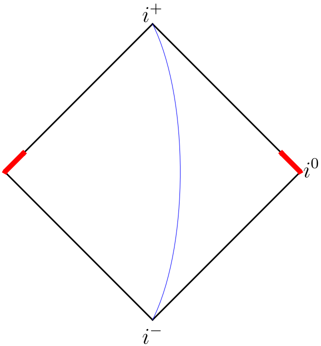
Proof of holography of information

$$
\begin{aligned}
A &= \sum_{n,m} c(n,m)|n\rangle\langle m| && \text{by step 1} \\
&\doteq \sum_{n,m} c(n,m) X_n |0\rangle\langle 0| X_m^\dagger && \text{by step 2} \\
&= \sum_{n,m} c(n,m) X_n \mathcal{P}_0 X_m^\dagger && \text{by step 3}
\end{aligned}
$$

So any operator, $\mathcal{H}^{(0)} \to \mathcal{H}^{(0)}$, can be approximated arbitrarily well by an operator near $\mathcal{I}^+_-$.



Massive particles



Previous analysis does not account for massive particles that go from i^- to i^+.

No satisfactory asymptotic description of massive fields at $\mathcal{I}^\pm$.

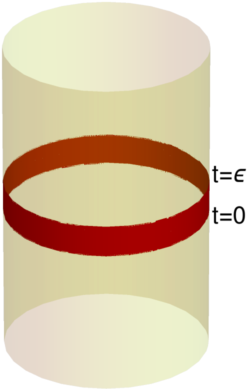
Some progress possible by studying massive particles at i^0.

[Laddha, Prabhu, S.R, Shrivastava, 2022]

[Anupam, Athira, Paul, S.R., (in progress)]

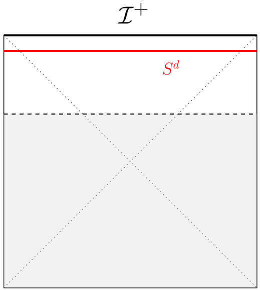
Holography of Information in AdS

Very similar arguments imply that all operators on the boundary of AdS can be represented in a small time band $[0, \epsilon]$. (No subtlety with massive particles.)



This is consistent with AdS/CFT but does not assume it.

dS analysis



$$\mathcal{H}\Psi[g,\chi] = 0; \qquad \mathcal{H}_i\Psi[g,\chi] = 0.$$

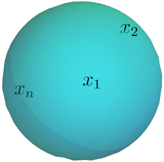
At late times, WDW equation forces

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

[Chakraborty, Chakravarty, Godet, Paul, S.R., 2023]

S is a universal phase. $Z[g,\chi]$ is a diff-and-almost-Weyl invariant functional that specifies the state.

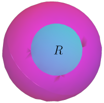
Cosmological correlators



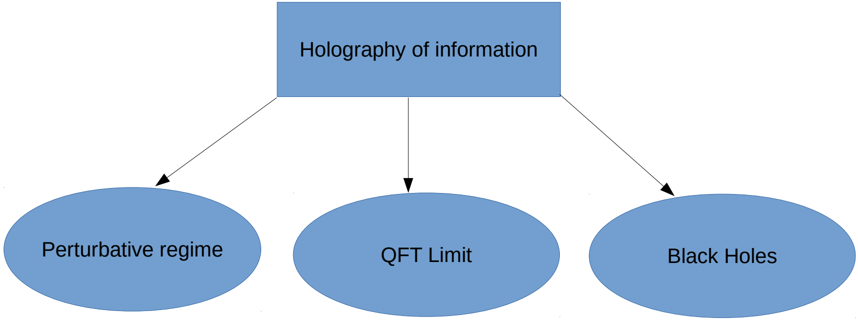
Cosmological correlators must be gauge fixed. They are labelled by local points but secretly nonlocal.

$$\langle\langle \Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\rangle_{CC} = \int |\Psi|^2\chi(x_1)\dots\chi(x_n)\delta(\text{g.f})\Delta'_{FP}DgD\chi$$

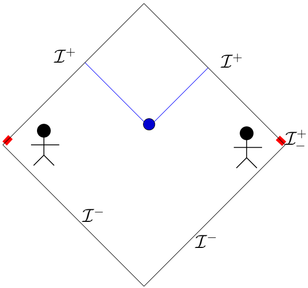
Constraints $\Rightarrow$ form of the wavefunctional $\Rightarrow$ symmetries of cosmological correlators.



Regimes


Holography of information
Perturbative regime
QFT Limit
Black Holes

Perturbative evidence



- Consider an excitation that hits null infinity in the retarded-time interval $[0, 1]$.

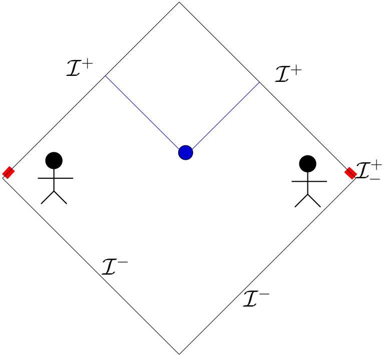
$$|f\rangle = e^{-i\lambda \int d^2\Omega \int_0^1 dx\, f(x,\Omega) O(x,\Omega)} |0\rangle.$$

- **Challenge:** Using perturbative quantum correlators near $\mathcal{I}^+_-$ determine $f(x, \Omega)$.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Chowdhury, Papadoulaki, S.R., 2020]

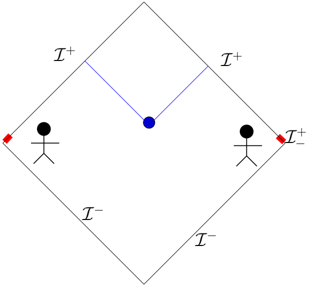
Perturbative evidence



$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = G\lambda \int_0^1 \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx + O\left(\lambda^2\right)$$

$$= -G\lambda \sum_{n=0}^{\infty} \frac{1}{u^{n+1}} \int_0^1 x^n f(x,\Omega')dx.$$

Significance of gravity



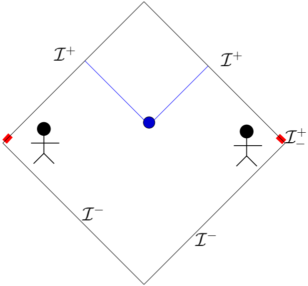
- This idea cannot work without gravity.

- Nongravitational gauge theories contain local gauge-invariant bulk operators.

$$|0\rangle \qquad \text{and} \qquad e^{i\mathrm{Tr}(F^2)(u=0)}|0\rangle$$

cannot be distinguished by any measurement near $\mathcal{I}^+_-$ without gravity.

Classical limit

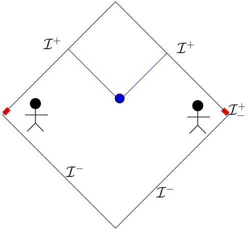


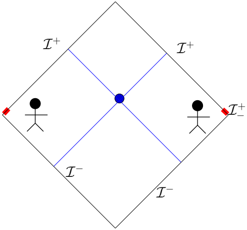
- Restoring factors of $\hbar$ and c

$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = \frac{\hbar G}{c^3}\lambda \int \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx + \mathrm{O}\left(\lambda^2\right).$$

- Protocol fails when $G \to 0$ or $\hbar \to 0$.

Locality



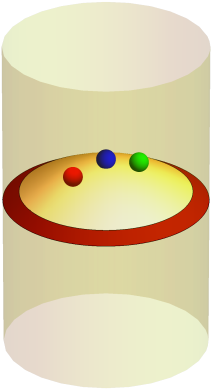


- Phenomenon visible in linearized theory; no complicated nonlocal dynamics.

- In gravity, after sending the excitation, it is impossible to erase information from infinity. (In nongravitational theories, this is possible).

AdS physical protocol

In AdS, we can set up a precise physical protocol for observers near infinity to identify a low-energy state.



In a low-energy state, $|g\rangle$.

$$1 - \| P_{E<\Lambda} |g\rangle \|^2 \ll 1,$$

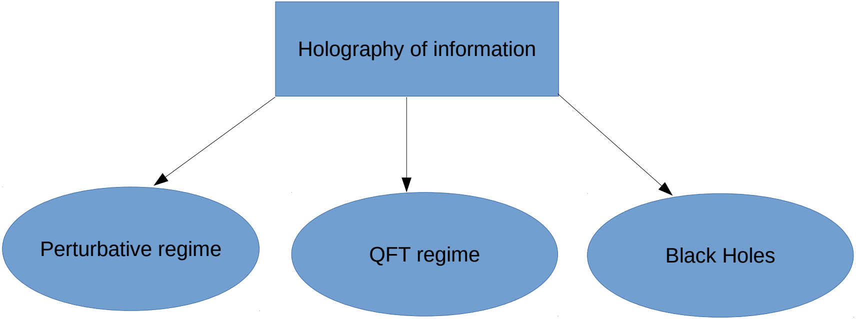
Observers can find $|g_{\text{est}}\rangle$ with

$$1 - |\langle g_{\text{est}} | g \rangle|^2 \leq \delta,$$

where $\delta \ll 1$ but $\delta \gg \frac{1}{N}$.

[Chowdhury, Papadoulaki, S.R., 2020]

Regimes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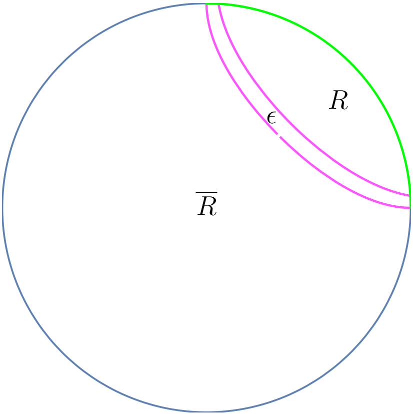
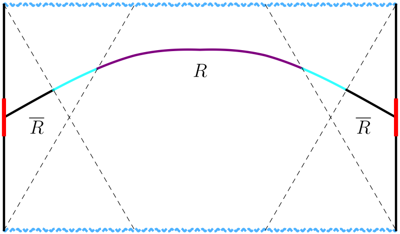
Two regions extending to infinity



When R and $\overline{R}$ both extend to infinity, the split property is obvious even if factorizing the Hilbert space is subtle due to edge effects.

(see talk by Thomas Mertens)

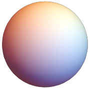
Double horizons



- All information on the boundary is available in a small band on the boundary.
- But some information in the bulk might never be available on the boundary.

(see talk by Vijay Balasubramanian)

Classical limit



Distinct spherically symmetric excitations with same total mass have identical gravitational fields at infinity (Birkhoff's theorem).

No holography of information in the classical limit.

[Folkestad, 2023]

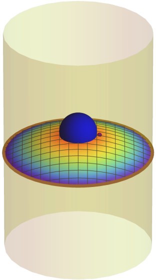
$\langle H^n \rangle$ distinguishes between such distributions in the quantum theory.

Coarse graining

- Consider a heavy state with many possible microstates.
- But restrict observations outside to accuracy less than $\mathrm{O}\left(e^{-S}\right)$ where S controls the density of states.
- Allows one to hide information in a region.

[Bahiru, Belin, Papadodimas, Sarosi, Vardian, 2022]

Observers



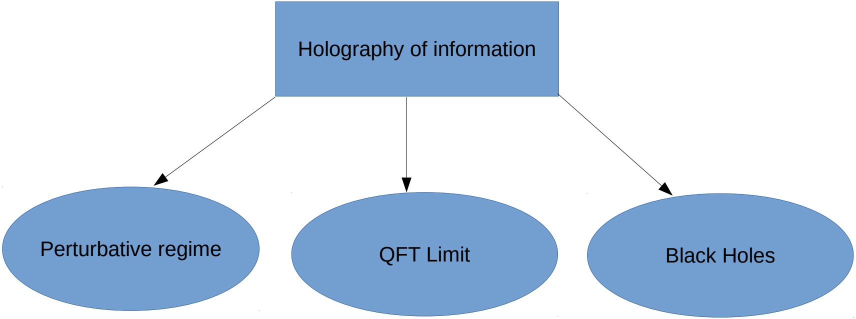
Introducing a heavy observer (with entropy S_{obs}) allows definition of approximately local algebras up to an accuracy $e^{-S_{\mathrm{obs}}}$.

[Chandrasekaran, Longo, Penington, Witten, 2022]

[Jensen, Sorce, Speranza, 2023]

[Jensen, Speranza, S.R., (in progress)]

Regimes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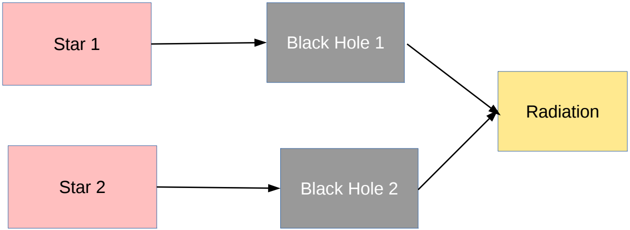
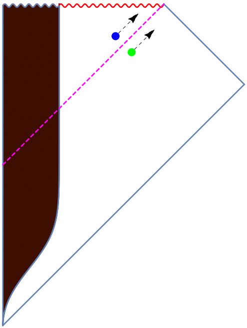
Information paradox



Hawking argued that since radiation is uncorrelated with the initial state, the time-evolution arrows cannot be reversed. This contradicts unitarity.

Significance of exponentially suppressed terms



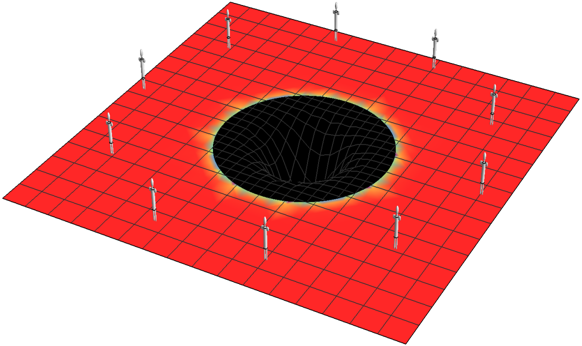
Typical pure states are exponentially close to mixed states

$$\langle\Psi|A|\Psi\rangle = \mathrm{Tr}(\rho_{\text{micro}} A) + O\left(e^{-\frac{S}{2}}\right)$$

[Lloyd, 1988]

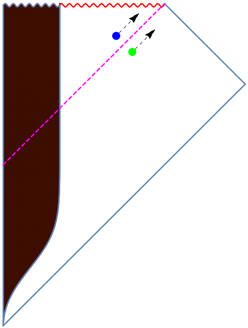
So to address the information paradox, we must allow discussion of exponentially precise observations at infinity.

Black-hole information



Holography of information implies that information about the black-hole microstate is always available outside with sufficiently precise observations.

Error in Hawking's argument?



Low point correlators

$$\langle a_\omega a_\omega^\dagger \rangle \approx \frac{1}{1 - e^{-\beta\omega}}$$

are insufficient to argue for information loss.

Where is the error in Hawking's argument?

One therefore has to introduce a hidden surface around each of these holes and apply the principle of ignorance to say that all field configurations on these hidden surfaces are equally probable provided they are compatible with the conservation of mass, angular momentum, etc. which can be measured by surface integrals at a distance from the hole.

Let H_1 be the Hilbert space of all possible data on the initial surface, H_2 be the Hilbert space of all possible data on the hidden surface, and H_3 be the Hilbert space of all possible data on the final surface. The basic assumption of quantum theory is that there is some tensor S_{ABC} whose three indices refer to H_3, H_2, and H_1, respectively, such that if

$$\xi_C \in H_1, \quad \zeta_B \in H_2, \quad \chi_A \in H_3,$$

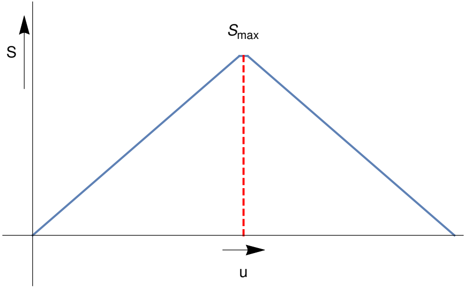
then

$$\sum \sum \sum S_{ABC} \chi_A \zeta_B \xi_C$$

is the amplitude to have the initial state ξ_C, the final state χ_A, and the state ζ_B on the hidden surface. Given only the initial state ξ_C one cannot determine the final state but only the element $\sum S_{ABC} \xi_C$ of the tensor product $H_2 \otimes H_3$. Because one is ignorant of the state on the hidden surface one cannot find the amplitude for measurements on the final surface to give the answer χ_A but one can calculate the probability for this outcome to be $\sum\sum\sum \rho_{CD} \bar{\chi}_C \chi_D$, where

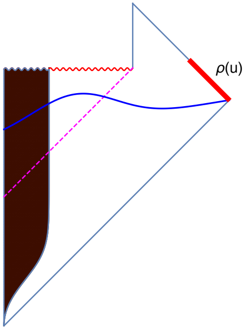
- Hawking assumed that the Hilbert space factorizes up to a few global constraints imposed by the no-hair theorem.
- Our previous discussion shows that this fails in quantum gravity.

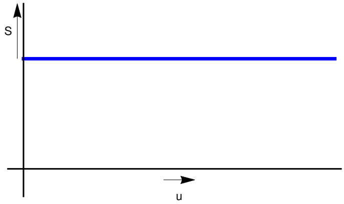
Inapplicability of the Page curve



The idea that the von Neumann entropy of the region outside follows the Page curve also assumes factorization.

von Neumann entropy on $\mathcal{I}^+$



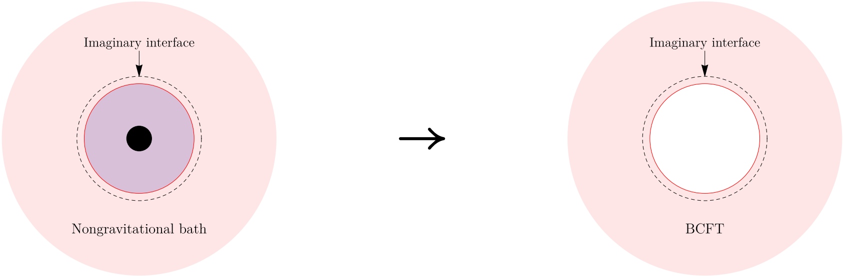


We can prove that the von Neumann entropy of a segment $(-\infty, u)$ of $\mathcal{I}^+$ is independent of u.

[Laddha, Prabhu, S.R., Shrivastava, 2020]

Other entropies do follow a Page curve (more below).

Nature of information transfer



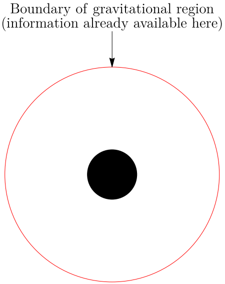
- Recent computations of the Page curve do not contradict our results.
- Page curve computes information transfer between nongravitating subregions. (Phrase: "Page curve of Hawking radiation" is misleading.)

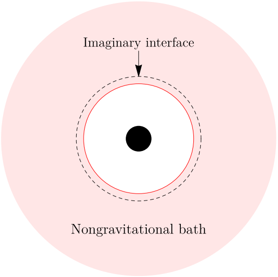
Nature of information transfer

Questions have been raised about how information “emerges” from the black holes.

[Martinec, 2022]

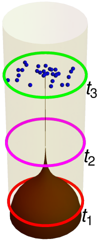
[Mathur, 2020–23]





Holography of information $\Longrightarrow$ information is present near the boundary; merely transferred to the nongravitational region.

Small black holes: difficulty of recovering information



Before BH formation, information available in low-point correlators

$$\langle T_{\mu\nu}(\Omega_1, t_1)\, T_{\mu\nu}(\Omega_2, t_2)\rangle$$

- At t_2, information requires S-pt correlators to accuracy e^{-S}

$$\langle T_{\mu\nu}(\Omega_1, t_1) \ldots T_{\mu\nu}(\Omega_S, t_S)$$

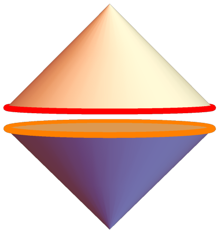
- At t_3, may require S'-pt correlators to accuracy $e^{-S'}$ where $S' > S$ is the entropy of Hawking radiation.

$$\langle T_{\mu\nu}(\Omega_1, t_1) \ldots T_{\mu\nu}(\Omega_{S'}, t_{S'})\rangle$$

(⚠ Caution: Analysis not watertight because small black holes are atypical states.)

Summary

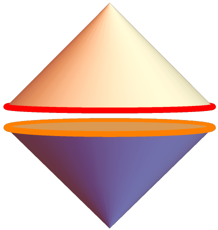
Standard theories of gravity store quantum information very differently from local quantum field theories.



- Nonperturbative result that relies on weak assumptions about the UV theory.
- For simple states, effect appears in perturbation theory.
- Sheds light on why gravitational theories are holographic.
- By coarse-graining observations about heavy states, we can make gravity behave like local QFT.

Summary

Accounting for this unusual localization of information resolves several paradoxes about black holes.



- Recent computations of the Page curve involve a nongravitational bath where the Hilbert space does factorize.
- Holography of information helps to understand how information enters the bath.