From Black Holes to Holographic Spacetime

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Outline

Review of thermodynamics, statistical mechanics, entropy

Entropy of black holes and entropy in quantum gravity

The holographic principle

The AdS/CFT Correspondence

General lessons about holographic dualities

Statistical Mechanics and Thermodynamics

Our discussion uses Statistical Mechanics and Thermodynamics, closely related branches of physics dealing with the behavior of matter, energy and their interactions, in large systems of particles.

Thermodynamics: focuses on the macroscopic properties of matter and energy; deals with concepts like temperature, pressure, volume, and energy transfer between systems. Gives a set of fundamental laws and principles describing how energy flows and how systems reach equilibrium. Useful for understanding heat engines, phase transitions, energy conversion.

Statistical Mechanics: delves into microscopic behavior of particles and their interactions to explain macroscopic properties of a system. Uses statistical methods and probability theory to connect behavior of individual particles to overall properties of the system. Statistical Mechanics aims to explain relationships between temperature, pressure, energy and other macroscopic variables in terms of the statistical distribution of microstates.

Relating Statistical Mechanics and Thermodynamics

Our discussion uses Statistical Mechanics and Thermodynamics, closely related branches of physics dealing with the behavior of matter, energy and their interactions, in large systems of particles.

Relation between Statistical Mechanics and Thermodynamics: is that of micro and macro descriptions. Thermodynamics provides empirically derived laws and principles for the behavior of macroscopic systems without the microscopic details. Statistical Mechanics offers a microscopic foundation for the thermodynamic principles. Connects the behavior of individual particles to bulk behavior of the system and explains why thermodynamic laws work as they do.

Statistical Mechanics is a bridge between the microscopic behavior of particles and observable macroscopic properties described by Thermodynamics. Explains how macroscopic quantities emerge from interactions of countless individual particles, adding a deeper layer of understanding to the macroscopic laws of Thermodynamics.

One way to understand this lecture

We know the laws of black hole thermodynamics.

Originally formulated based on analogies between results derived using general relativity's description of black holes and ordinary thermodynamics.

Supported by Hawking's groundbreaking work which introduced the concept that black holes can emit radiation due to quantum effects near their event horizons.

This Hawking radiation, led to the idea that black holes have a temperature and an associated entropy.

Using these laws of black hole thermodynamics can we get an understanding of the statistical mechanics of black holes?

Entropy

Microstate: a specific arrangement or configuration of particles and energy in a system. It's a detailed snapshot of positions, velocities, and energy levels of all individual particles that make up the system.

Macrostate: describes macro properties of a system without specifying the exact positions and energies of individual particles. Includes information like temperature, pressure, total energy and volume.

Many microstates (possible arrangements of particles and energy) lead to same macrostate (overall observed properties). Some macrostates have more corresponding microstates than others. A macrostate with more associated microstates is more probable and has higher entropy. A macrostate with fewer corresponding microstates is less probable and has lower entropy.

Entropy is related to the number of ways particles and energy can be arranged to achieve a certain macro state. More ways to arrange them mean higher entropy, while fewer ways mean lower entropy.

Entropy

The Boltzmann distribution gives the probability that a system is in a certain state as a function of the state's energy and the temperature of the system

$$p_i = \frac{e^{-\beta E_i}}{Z}$$

where β is the inverse temperature $\beta = T^{-1}$ and partition function Z is a normalization

$$Z = \sum_i e^{-\beta E_i}$$

State E_i has degeneracy g_{E_i}. The energy E_i is a macrostate label while the index i is a microstate label. g_{E_i} microstates have energy E_i.

$$Z = \sum_{E_i} g_{E_i} e^{-\beta E_i} = \sum_{E_i} e^{-\beta\left(E_i - T\log(g_{E_i})\right)}$$

$S_{E_i} = \log(g_{E_i})$ is the *entropy* of macrostate with energy E_i. The free energy of the macrostate is

$$F_{E_i} \equiv E_i - T\log(g_{E_i}) = E_i - TS_{E_i}$$

Entropy and magnetism

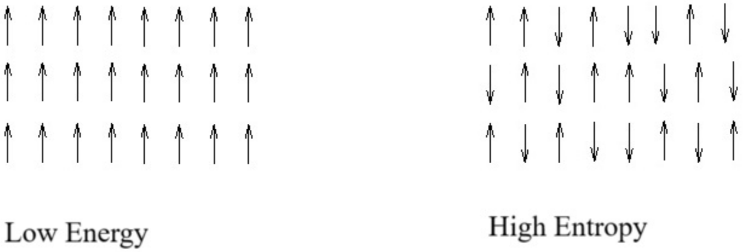
In terms of the free energy

$$Z = \sum_{E_i} e^{-\beta F_{E_i}}$$

This sum is dominated by the terms that minimize the free energy F_{E_i}.

$$F_{E_i} = E_i - TS_{E_i}$$

At low temperature the second term can be neglected and we minimize the energy. At high temperature the first term can be neglected and we maximize the entropy.



Maximum Possible Entropy: A Counting Problem

The maximum possible entropy, for a system in a volume V is obtained if we simply declare there is only one macrostate.

In this case, for a quantum system (for example) the maximum possible entropy is simply given by the logarithm of the number of distinct orthogonal quantum states.

So the computation of the maximum possible entropy reduces to counting the total number of distinct states.

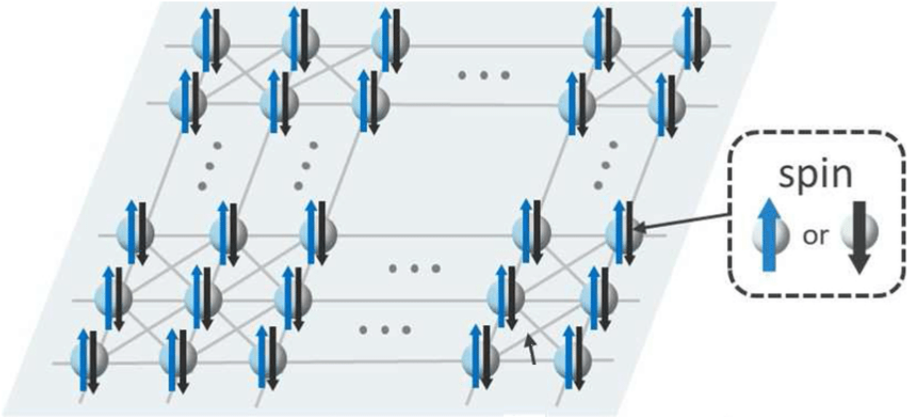
For a lattice composed of n sites, with a spin on each site the number of states is

$$N_{\rm states} = 2^{n_{\rm sites}}$$

where each spin can be in one of two states. The maximum possible entropy is then

$$S_{\rm max} = n_{\rm sites} \log 2$$

Entropy is Extensive



Maximum entropy is $S_{\max} = n \log 2$. The number of lattice sites n is proportional to volume of the region: $S_{\max} \propto V$. Holds for any theory in which the laws of nature are reasonably local.

Entropy of black holes

The second law of thermodynamics predicts that the entropy of isolated systems left to spontaneous evolution can't decrease. Isolated systems arrive at a state of thermodynamic equilibrium where the entropy is highest for the given energy.

The second law of thermodynamics requires that black holes have entropy. Otherwise we can violate the second law by throwing mass into the black hole. The increase of entropy of the black hole must more than compensate for the decrease of the entropy carried by the object that was swallowed. For this reason, in 1972, Bekenstein conjectured that black holes must have an entropy proportional to the area of their horizon.

In 1974, Hawking confirmed Bekenstein's conjecture and fixed the constant of proportionality

$$S = \frac{A}{4G}$$

This seems to be in tension with the observation that entropy is extensive. Can we produce a sharp paradox?

Entropy in quantum gravity: A paradox

Suppose a region of space of volume V does not contain enough energy to form a black hole.

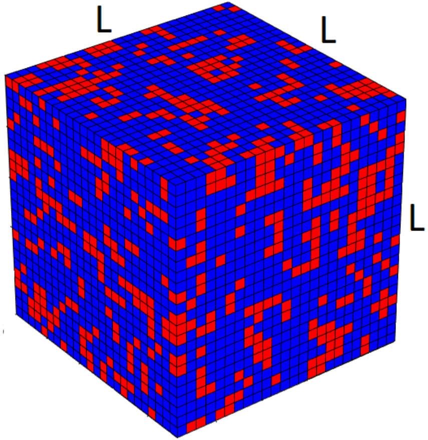
Further, computing this entropy using "common sense" we would find an entropy proportional to V and so greater than the entropy of a black hole which is just big enough to fit into V.

By throwing in additional energy we would now form a black hole.

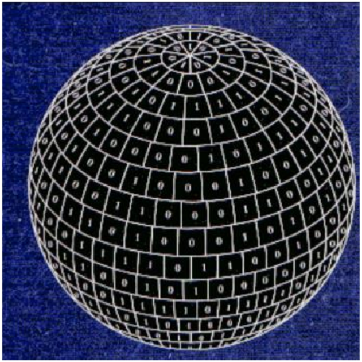
Since the entropy of the black hole is smaller than the original entropy, we violate the second law of thermodynamics.

There must be an error in our argument!

Entropy in quantum gravity: A paradox



$$S \propto L^3$$



$$S \propto L^2$$

Figure: Evidently we were mislead by "common sense" and the entropy of the original system was not proportional to V, but rather it is proportional to the area of the boundary of V.

The Holographic Principle

Proposed by Gerard 't Hooft and Leonard Susskind.

The holographic principle is a conjectured property of quantum gravity that states that the description of a region of spacetime in quantum gravity is described using a non-gravitational theory encoded on the lower-dimensional boundary of the region.

A quote from Leonard Susskind: "The three-dimensional world of ordinary experience - the universe filled with galaxies, stars, planets, houses, boulders, and people - is a hologram, an image of reality coded on a distant two-dimensional surface."

This sounds fantastic - is it true? The prime example of holography is the AdS/CFT correspondence.

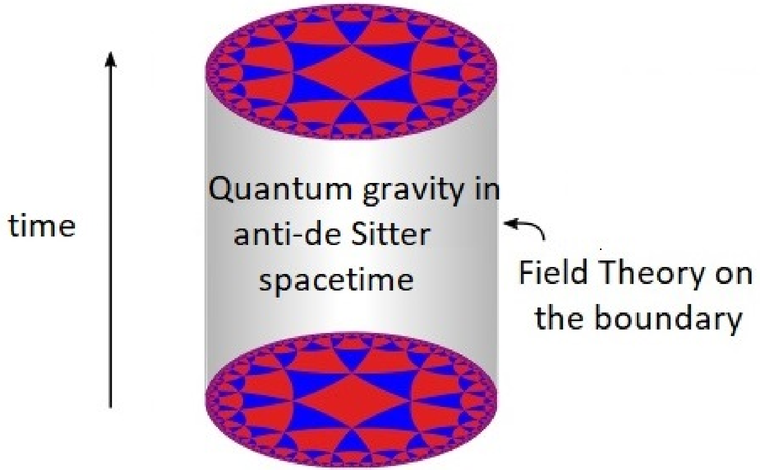
The AdS/CFT Correspondence

The most famous example of the AdS/CFT correspondence states that: Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N} = 4$ supersymmetric Yang–Mills theory on the four-dimensional boundary. This is a detailed realization of the holohraphic principle.

The spacetime on which gravity lives (AdS_5) is five-dimensional. There are five additional compact dimensions (S^5). Anti-de Sitter spacetime has a negative cosmological constant. The real world has a positive cosmological constant and spacetime is four-dimensional (at least macroscopically) so this version of the correspondence does not provide a realistic model of gravity.

Supersymmetric Yang-Mills theory is not a viable model of any real-world system as it assumes a large amount of supersymmetry. However, the boundary theory shares some features in common with quantum chromodynamics, the fundamental theory of the strong force.

The AdS/CFT Correspondence

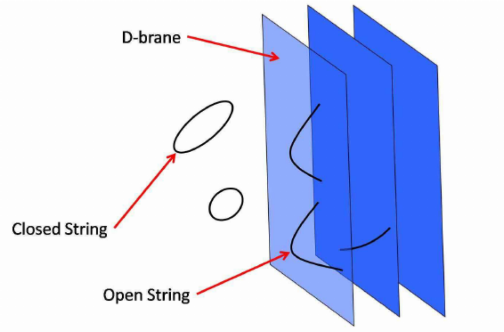


The initial and final time slices of the Anti de-Sitter spacetime are shown.

String Theory

In string theory, the basic building blocks are one-dimensional strings rather than point particles. The strings can be either open or closed.

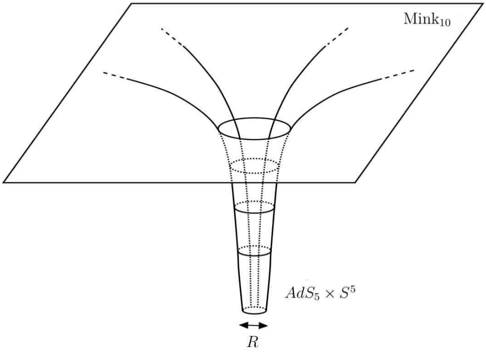
Open Strings: have two endpoints which end on an object known as a *D*-brane. They can vibrate and move in space. Open strings vibrations correspond to particle states giving the non-gravitational interactions, described by non-Abelian gauge theories.



Closed Strings: have no endpoints. Closed strings can vibrate and interact along their entire length. Closed strings vibrations correspond to particle states giving the gravitational interactions.

D-branes as solitons

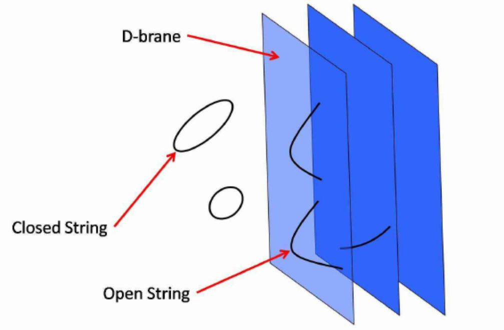
AdS CFT was discovered by thinking about two different ways to describe a stack of *D*-branes in string theory.



The stack of *D*-branes is described as a soliton (a stable non-spreading lump of energy) in a theory of closed strings. We can talk about the geometry of the spacetime as well as the fluxes sourced by any conserved charges. This description is entirely in terms of closed strings.

D-branes as defects

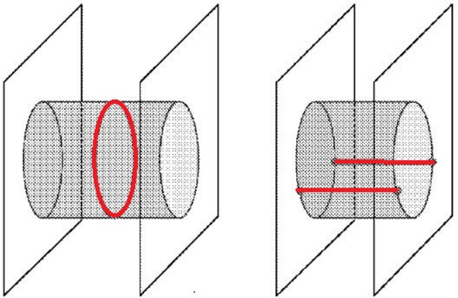
The second way to describe the stack of D-branes uses open and closed strings.



Each D-brane can absorb or emit closed strings. Open strings stretch between difference branes in the stack.

Open/closed string duality

Why do we believe these are two equivalent descriptions of the same object?



Using open/closed string duality Polchinski argued that D-branes carry the same electric and magnetic Ramond-Ramond charges as solitons in closed string theory. In other words, these two descriptions are different descriptions of the same physical system.

What is a low energy state?

Strings: The string has a massless mode (the string is not vibrating) separated by a gap from an infinte tower of massive excited string states (string is vibrating). At low energy we only access string states which are not excited. (Assume string tension is not zero!)

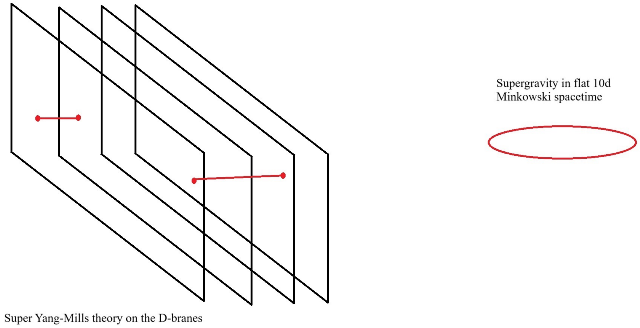
Particles: By quantum mechanics and relativity:

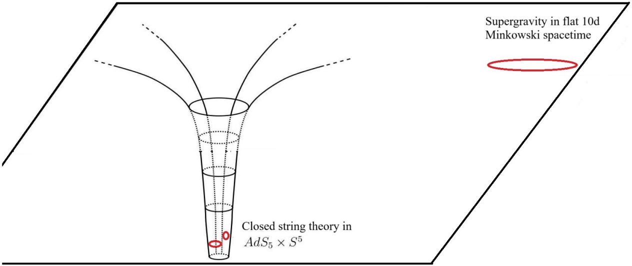
$$p = \frac{h}{\lambda} \qquad\qquad E^2 = m^2c^4 + p^2c^2 \tag{1}$$

Small E means $m = 0$ and small p. Small p means long wavelength. Low energy limit will correspond to very long wavelength supergravity modes.

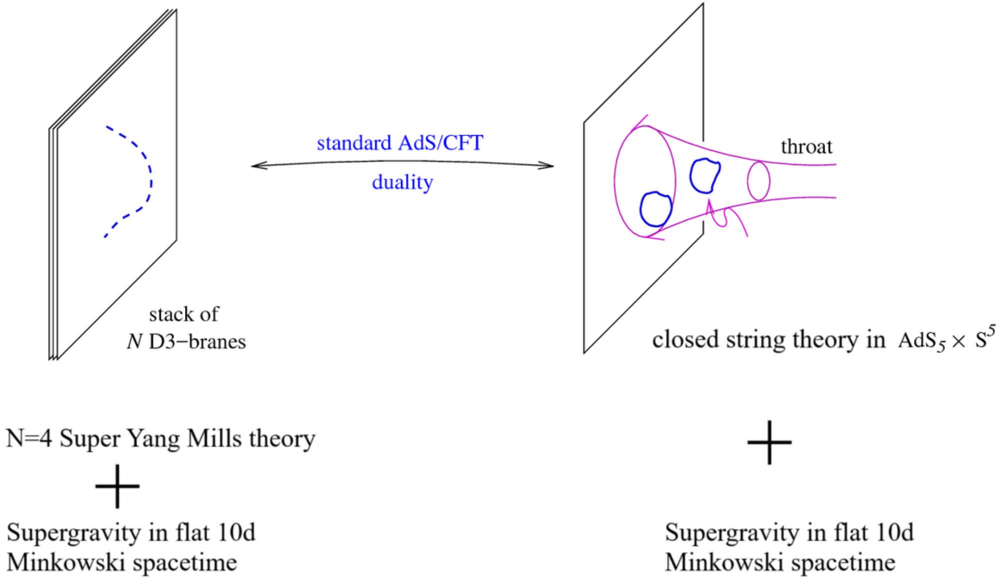
Objects in a potential energy well: If the potential energy well is deep enough, everything at the bottom of the well (string states excited or not as well as any other state) is a low energy state.

What is a low energy state?





Maldacena's decoupling limit



Basic Dictionary

In the 't Hooft limit of the super Yang-Mills theory with gauge group U(N)

$$N \to \infty \qquad g_{YM}^2 N = \lambda = \text{fixed}$$

In this limit the mapping between string theory and field theory parameters is

$$\frac{L^4}{\alpha'^2} = g_{YM}^2 N = \lambda \qquad g_{st} \sim \frac{1}{N}$$

where L sets the radius of curvature of the AdS spacetime.

When the field theory is strongly coupled (large λ) and at large N, the string theory is weakly coupled (small g_{st}) and the spacetime geometry is weakly curved. Many questions about strongly coupled large N super Yang-Mills theory can be answered using classical supergravity!

This is also a curse: the two sides are never simple for the same range of parameters.

Precision Tests of AdS/CFT

An important observable in any CFT are the scaling dimensions of it's primary operators

$$\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\rangle = \frac{\delta_{\Delta_1\Delta_2}}{|x_1-x_2|^{2\Delta_1}}$$

The operators of the CFT also obey an operator product algebra

$$\mathcal{O}_{\Delta}(x_1)\mathcal{O}_{\Delta_2}(x_2) = \sum_{\Delta} \lambda_{\Delta_1\Delta_2\Delta} C(x_1-x_2,\partial_{x_1})\mathcal{O}_{\Delta}(x_1)$$

The scaling dimensions Δ_i and the OPE coefficients $\lambda_{\Delta_1\Delta_2\Delta}$ depend on the dynamics of theory and could be can in a loop expansion using Feynman diagrams.

The complete set of primary operators, their dimensions and their OPE coefficints is a complete definition of the theory. Any correlation function of the theory can be computed with this *CFT data*.

From the point of view of AdS/CFT operator dimensions are also interesting: they map into energies of states in the dual gravity theory.

Precision Tests of AdS/CFT: Planar Integrability

The spectrum of scaling dimensions of primary operators can be obtained by diagonalizing the dilatation operator of the CFT.

This problem can actually be solved in the planar limit of the $\mathcal{N} = 4$ super Yang-Mills theory.

Planar $\mathcal{N} = 4$ super Yang–Mills theory and free IIB superstrings on $AdS_5 \times S^5$ are exactly dual by the AdS/CFT correspondence.

Remarkably, the Dilatation operator is equal to the Hamiltonian of an integrable spin chain. Using the powerful methods of integrability the spectrum of anomalous dimensions can be computed to all orders in the 't Hooft coupling λ.

The dynamics of the dual string is also integrable and a complete and detailed matching with the CFT results has been achieved.

Duality between Higher Spin Gravity and Vector Models

A simple holographic duality discovered by Klebanov and Polyakov claims a correspondence between (Vasiliev's) higher spin gravity theory in AdS_4 and $O(N)$ vector models.

Perhaps simplest, nontrivial example of AdS/CFT:

1. Spectrum of operators in CFT not renormalized at infinite N (Giombi, Minwalla, Prakash, Trivedi, Wadia, et al. arXiv:1110.4386), and
2. Spectrum of fields in the bulk theory is simple. talk about spectrum of bulk fields.(Vasiliev, arXiv:hep-th/0106149 [hep-th])

Nontrivial content of duality (at least in perturbation theory) is in agreement between correlation functions in the bulk higher spin gauge theory and boundary vector model CFT. Giombi, Yin, arXiv:0912.3462 [hep-th] demonstrated match for free and critical $O(N)$ vector models.

A Simple Toy Model

For this example, at large N we can explain how the dual gravity theory is constructed starting from CFT.

This is achieved by giving a holographic map constructed by:

1. Reducing gravity to physical and independent degrees of freedom.
2. Reducing CFT to its independent degrees of freedom.
3. Identifying the complete set of degrees of freedom of CFT with corresponding degrees of freedom in gravity.

The mapping illustrates general expectations of holography.

Counting degrees of freedom

In Higher Spin Gravity: Fix light cone gauge: $A^{+\mu_2\cdots\mu_{2s}} = 0$. Solve the gauge constraint: eliminates $A^{-\mu_2\cdots\mu_{2s}}$. We are left with a traceless and symmetric rank-$2s$ tensor $A^{a_1 a_2\cdots a_{2s}}$ whose indices take 2 values X, Z. This has 2 independent components.

In CFT: Spinning currents are symmetric, traceless, conserved rank $2s$ tensors $J_{\mu_1\cdots\mu_{2s}}$.

There are $\frac{(2s+1)(2s+2)}{2}$ symmetric rank $2s$ tensors.

There are $4s+1$ symmetric, traceless rank $2s$ tensors.

There are 2 symmetric, traceless, conserved rank $2s$ tensors.

The number of independent components of the spinning primary currents match the number of physical and independent components of the gauge field.

We reduce the gravity theory to its independent and physical fields. To construct the holographic map we reduce the CFT to its independent fields, and then, at each $2s$ match the 2 components of the gauge field to the 2 components of the spinning current.

A Few of the Details

The holographic map follows in two steps

1. a change to gauge invariant (bilocal) field variables in the CFT.

$$\sigma(t,\vec{x}_1,\vec{x}_2) \quad = \quad \phi^a(t,\vec{x}_1)\phi^a(t,\vec{x}_2)$$

2. a change of spacetime coordinates.

Motivation for step 1: Loop expansion parameter of the CFT is $\hbar$. The loop expansion parameter of the dual gravity is $1/N$. After changing to invariant (bilocal) variables loop expansion parameter is $1/N$ matching the loop expansion parameter of the dual gravity.

Motivation for step 2: The bilocal transforms in a tensor product. The complete collection of higher spin fields transform in a direct sum. The natural change of basis

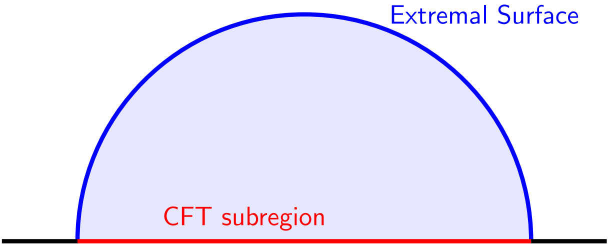
$$V_{[\frac{1}{2},0]} \otimes V_{[\frac{1}{2},0]} \longrightarrow V_{[1,0]} \ \oplus \ \bigoplus_{s=2,4,\cdots} V_{[s+1,s]}$$

determines a map between CFT and bulk coordinates.

Tests of the Holographic Mapping

There are a number of tests that this mapping passes. Here are two of them:

1. The reconstructed bulk gravity fields obey the correct bulk equations of motion with the correct boundary conditions, read from the GKPW dictionary.
2. The map recovers the entanglement wedge reconstruction expected of subregion duality. Using bilocals with both feet in the red interval we do indeed reconstruct all bulk field up to the RT surface.



Final Remarks

1. The holographic principle is a profound new principle of nature. Gravity and spacetime emerge from a lower dimensional non-gravitational theory.
2. The AdS/CFT correspondence gives us a fascinating concrete example of the principle. It also realizes an old idea that non-Abelian gauge theories are described by theories of strings.
3. As a real world application of these ideas it would be fascinating to work out the string theory dual to QCD, and to make genuine progress with the strong coupling dynamics of low energy QCD.
4. Black holes have played an important role in developing the holographic principle. However, their description using the tools of holography and AdS/CFT is largely still to be developed.
5. Doing gravity in Ant-de Sitter spacetime is doing gravity in a box. It is not yet clear how to use these lessons to describe gravity in our expanding universe, which is much better described as a de Sitter spacetime.

Thanks for your attention!