Spherical circle coverings and bubbles in foam

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Contents

COVERING THE SPHERE WITH EQUAL CIRCLES

The covering problem

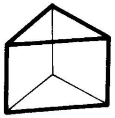
How must a sphere be covered by *n* equal circles (spherical caps) without interstices so that the angular radius of the circles will be as small as possible?

Known solutions in the 1980's

- Mathematically proven solutions for $n = 2, 3, 4, 5, 6, 7, 10, 12, 14$
- Conjectural solutions for $n = 8, 9, 32$

Known solutions in the 1980's

n = 2, 3, 4, 6, 12 L. Fejes Tóth
n = 5, 7, 8 K. Schütte
n = 9 E. Jucovič
n = 10, 14, 32 G. Fejes Tóth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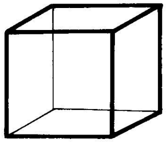
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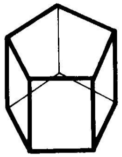
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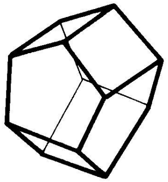
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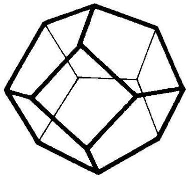
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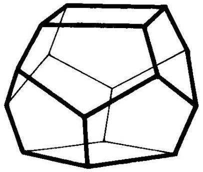
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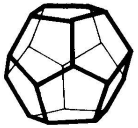
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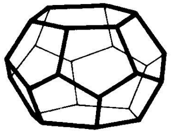
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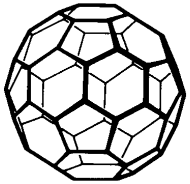
15 16

17 18 19 20 …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SOAP FILMS AND BUBBLES IN FOAM

Minimal nets formed by *n* soap-film-like cones

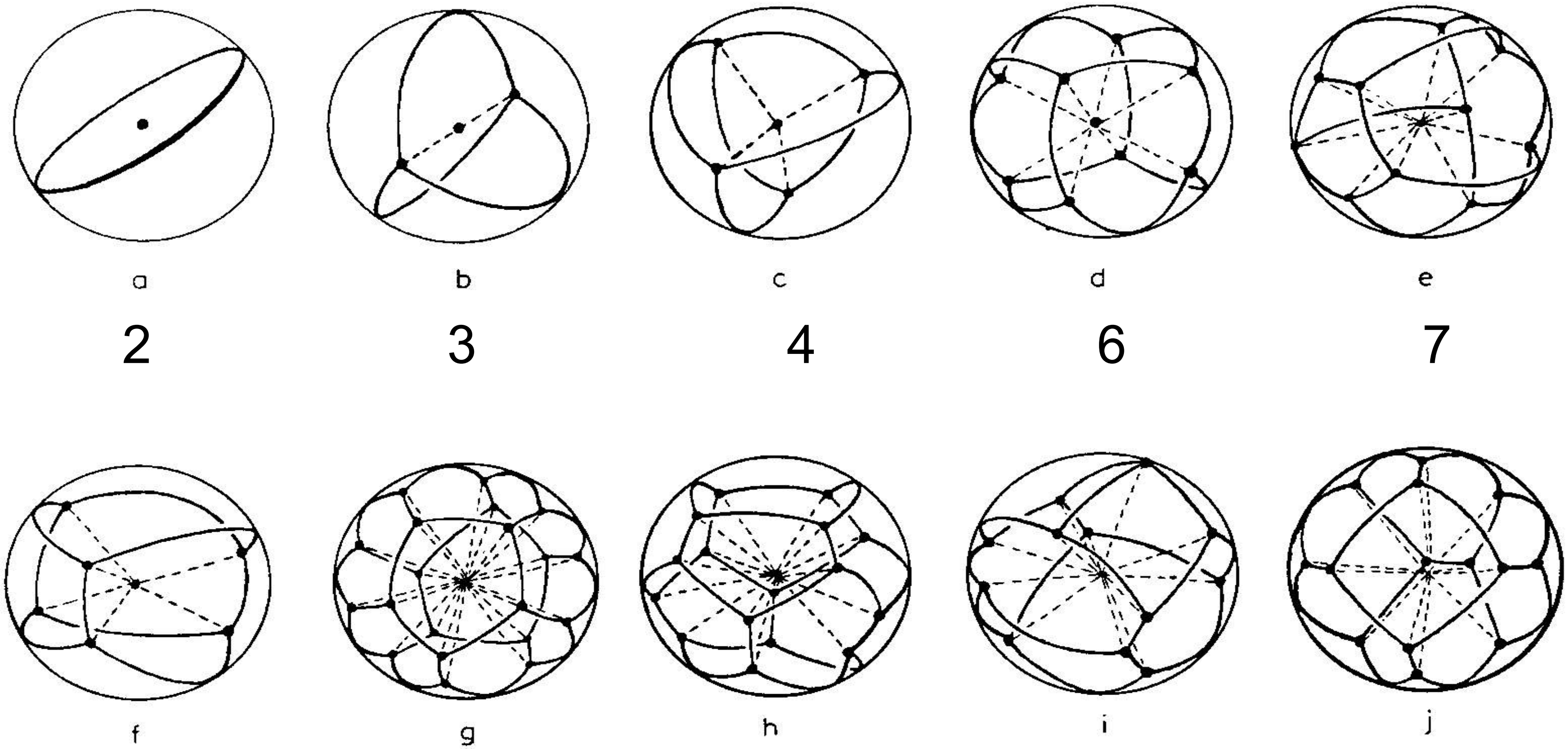


Fig. 1. The ten possible minimal one-dimensional nets formed by the intersection of a film cone by a sphere.

5 12 10 8 9

A. Heppes 1964, F.J. Almgren & J.E. Taylor 1976

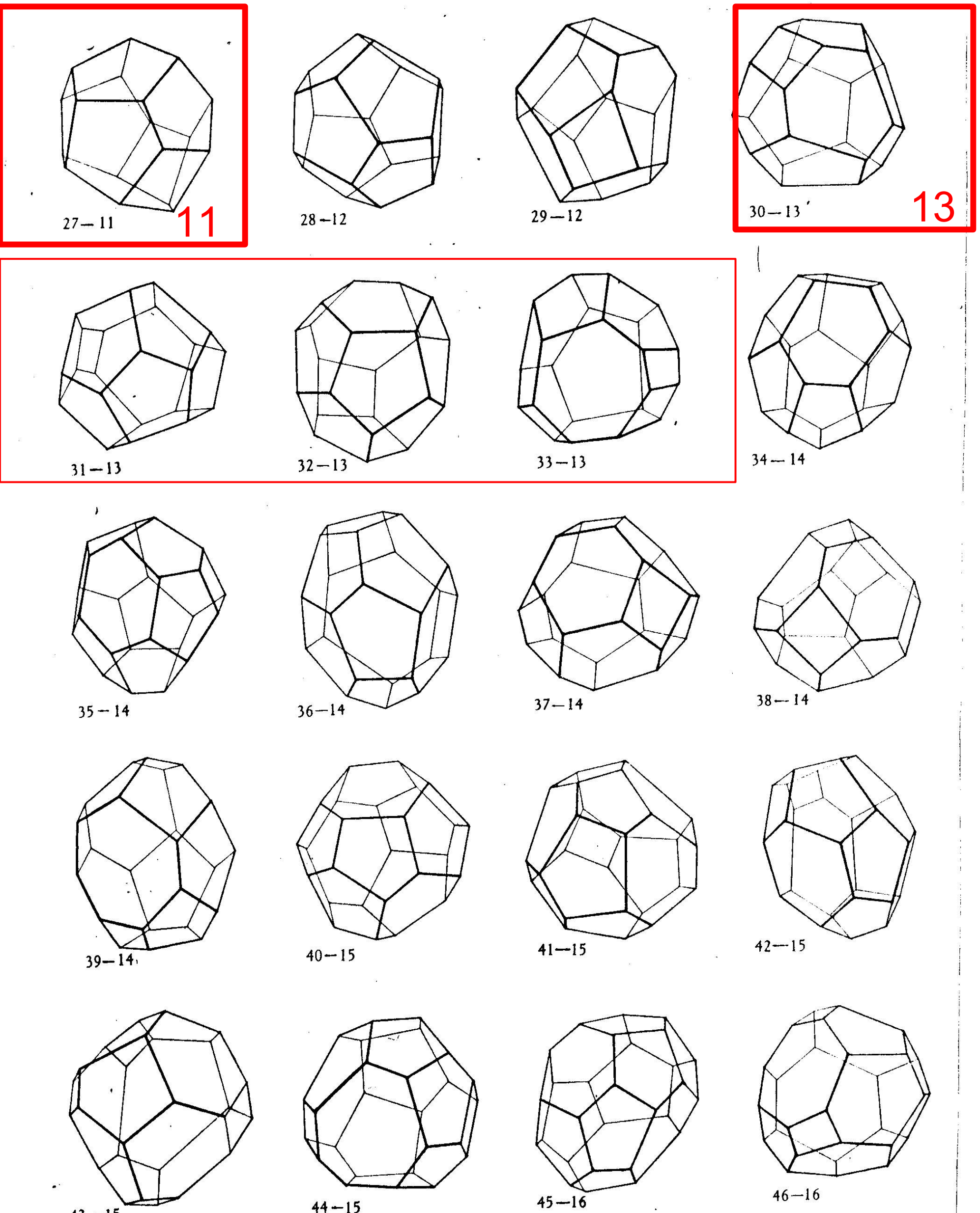


Fig. 27–46. Camera lucida drawings of central bubbles. × c. 4.4. The second number under each figure indicates the total number of faces. Curvatures of faces and edges are not shown in the drawings. Quadrilateral faces are designated as Q, pentagonal as P, hexagonal as Hx, heptagonal as Hp. The combinations of faces in each are as follows:—Fig. 27. 11 faces, 3 Q, 6 P, 2 Hx.—Fig. 28. 12 faces, 12 P.—Fig. 29. 12 faces, 4 Q, 4 P, 4 Hx.—Fig. 30. 13 faces, 1 Q, 10 P, 2 Hx.—Fig. 31. 13 faces, 2 Q, 8 P, 3 Hx.—Fig. 32. 13 faces, 3 Q, 6 P, 4 Hx.—Fig. 33. 13 faces, 3 Q, 7 P, 2 Hx, 1 Hp.—Fig. 34. 14 faces, 12 P, 2 Hx.—Fig. 35. 14 faces, 1 Q, 10 P, 3 Hx.—Fig. 36. 14 faces, 2 Q, 8 P.

Soap bubbles in Matzke's observations

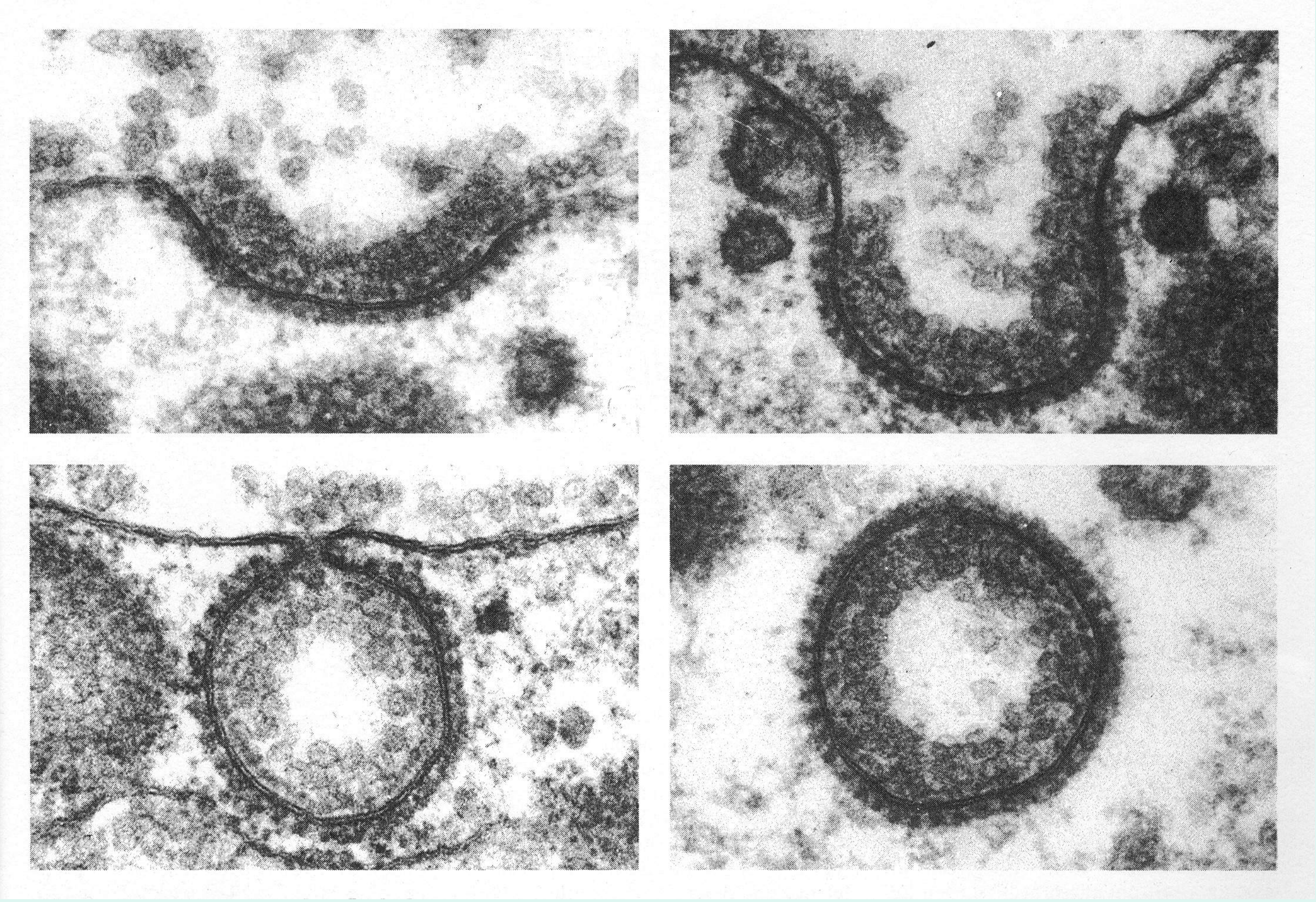
Minimum covering:

n = 11 T.T. & Zs. Gáspár 1986

n = 13 T.T. & Zs. Gáspár 1985

COATED VESICLES

Clathrin-dependent endocytosis



Formation of coated vesicles

Structure of coated vesicles

Barbara Pearse 1975

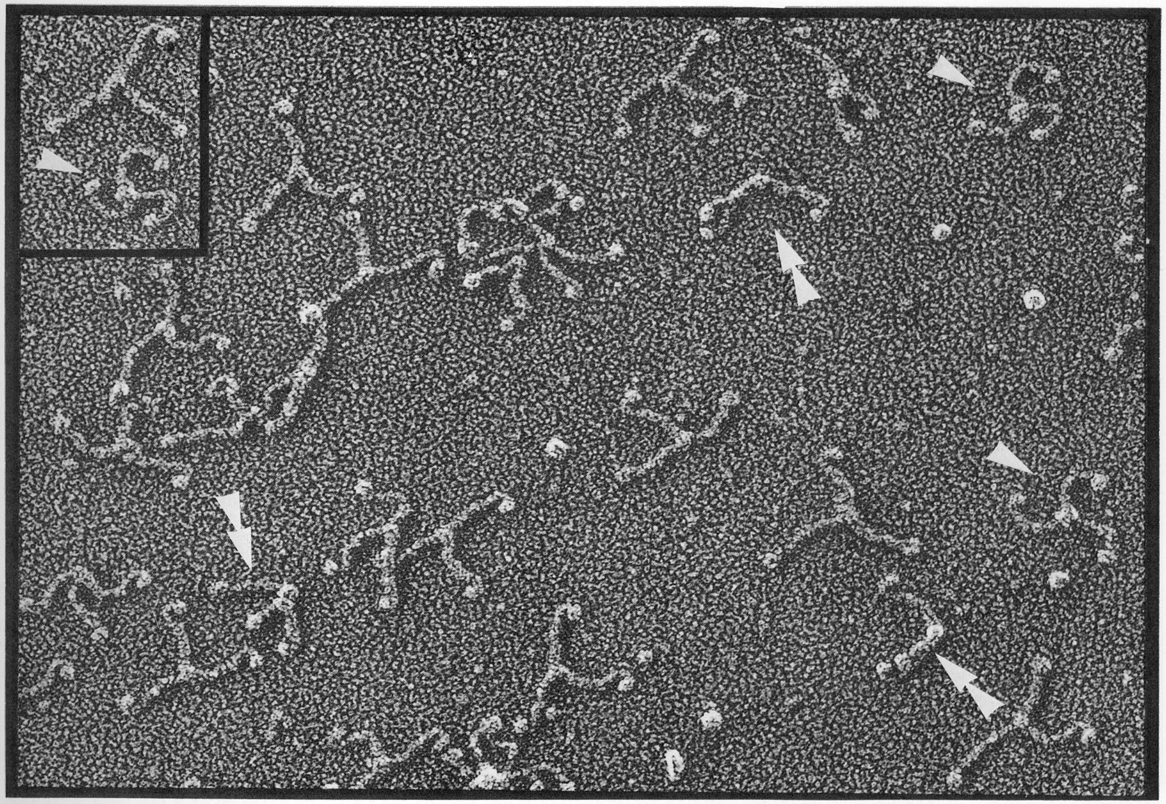


FIG. 4. Clathrin trimers maintained in 100 mM TEA buffer, pH 8.0, during exposure to mica, illustrating the predominantly counterclockwise orientation characteristic of adsorption under such high-affinity conditions. Some molecules still land in the clockwise "triskelial" orientation (arrowheads); a few even land on edge to give a side view of what may have been their original "pucker" (double arrowheads). × 230 000.

Clathrin triskelions

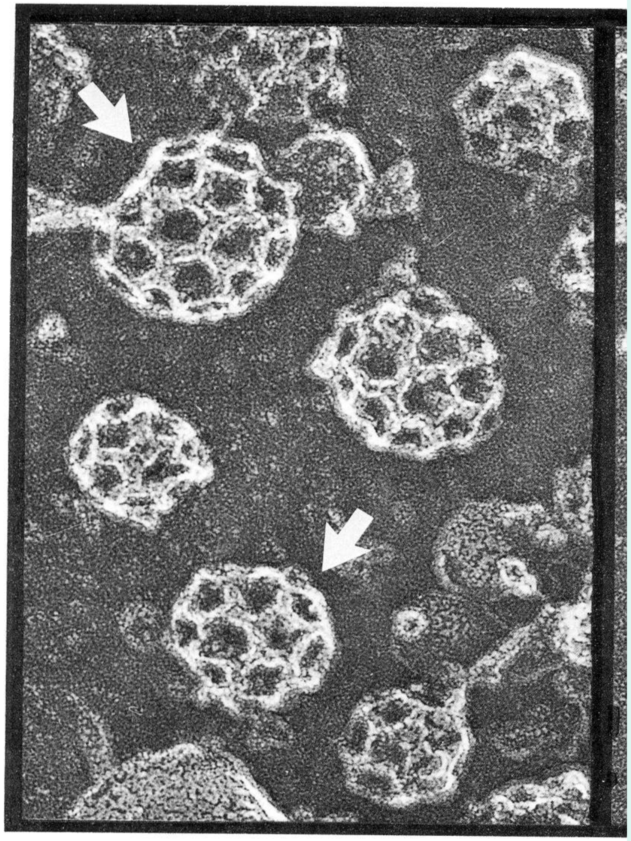
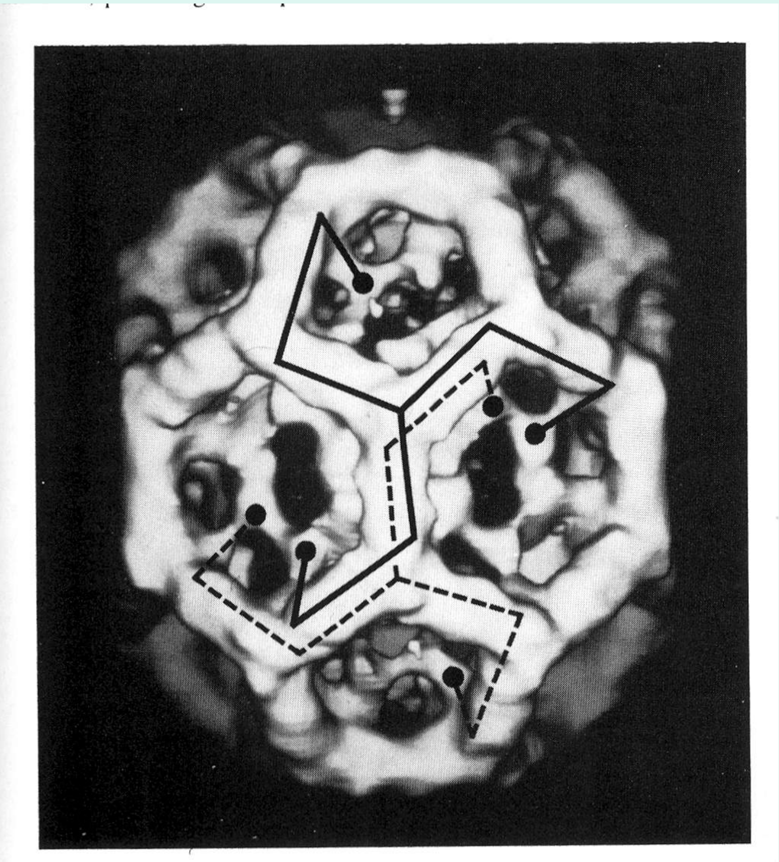


FIG. 11. Stereo view of the above coated vesicle prepara
rrow); a slightly smaller basket of "intermediate" configura

Clathrin basket coat

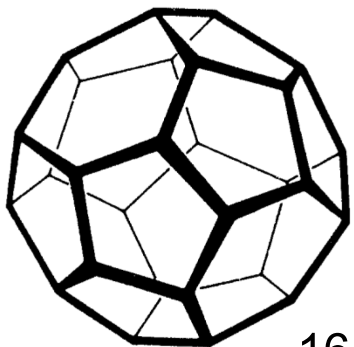
Coated vesicle



Clathrin basket

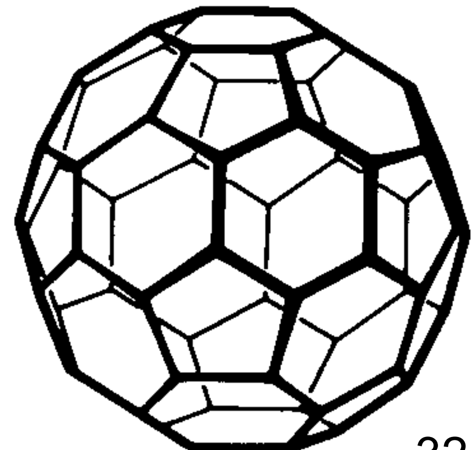
Street lamp in Kyoto

Identified clathrin baskets



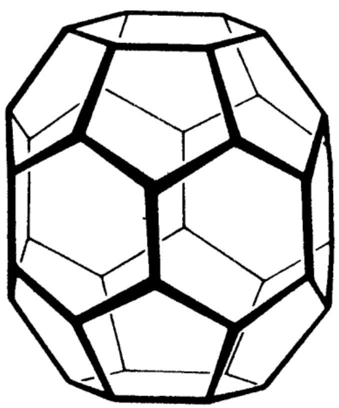
16

tetrahedral



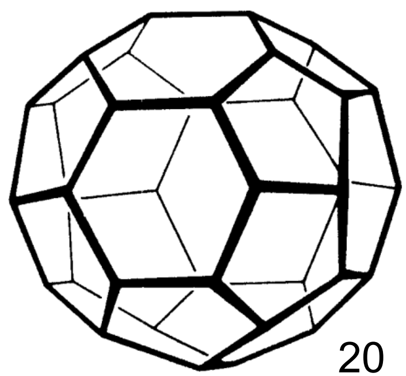
32

soccer ball



20

barrel



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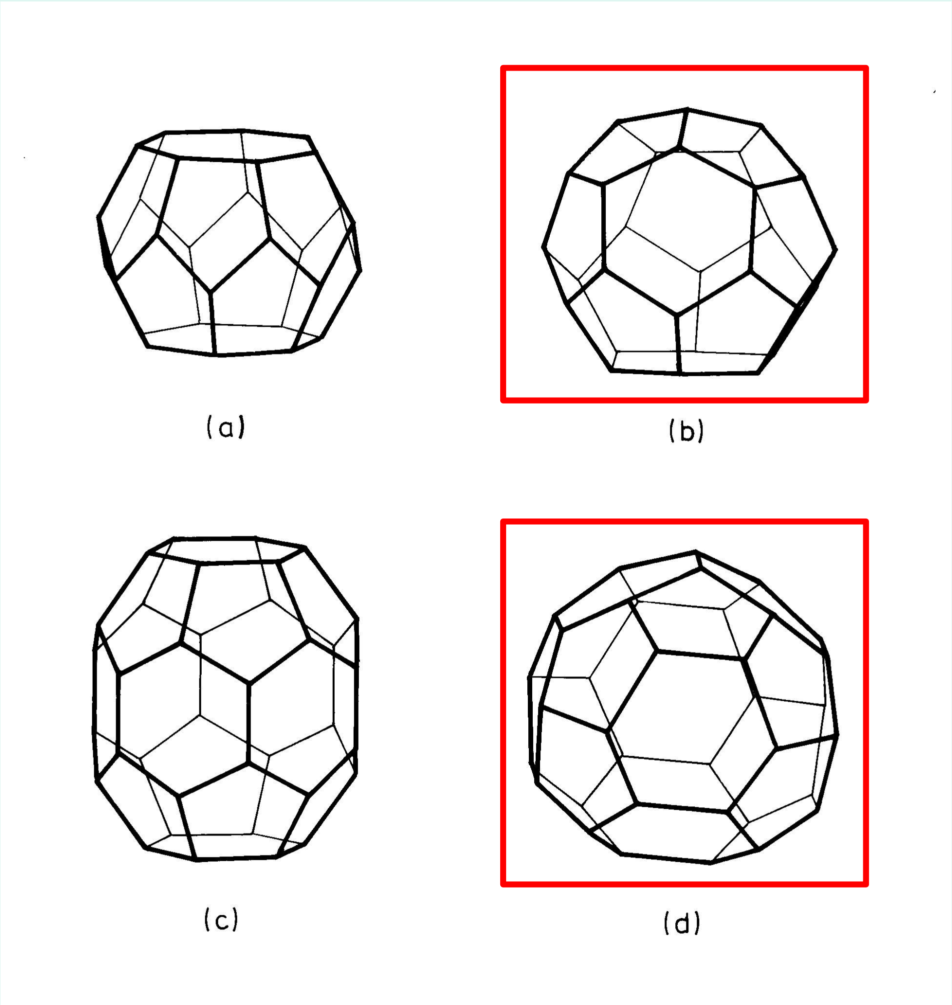
tennis ball

Circle coverings based on identified clathrin baskets

n = 16

n = 20



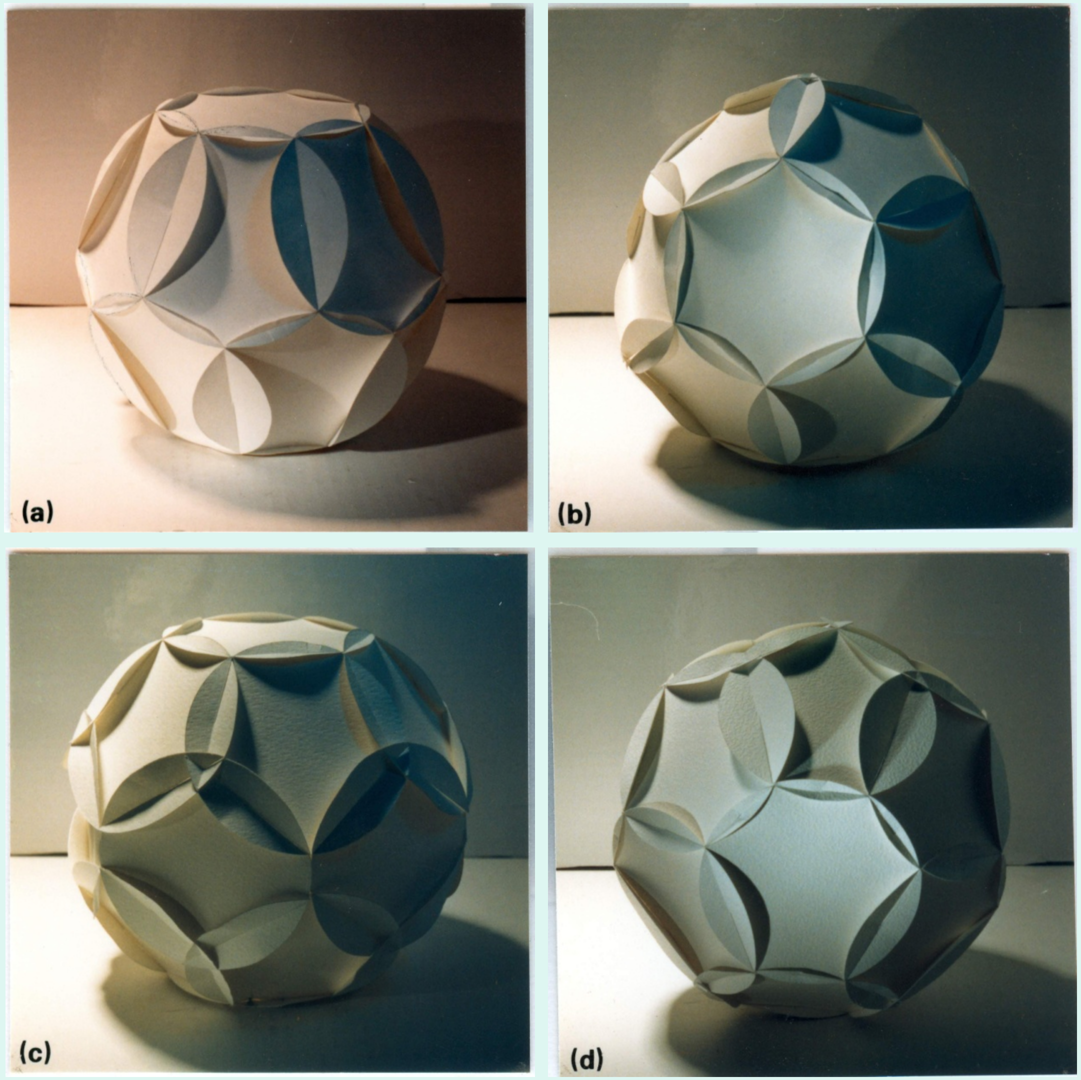
n = 16

n = 20

Circle coverings based on identified clathrin baskets

n = 16

n = 16



n = 20

n = 20

n = 16 and 20 T.T. & Zs. Gáspár 1991

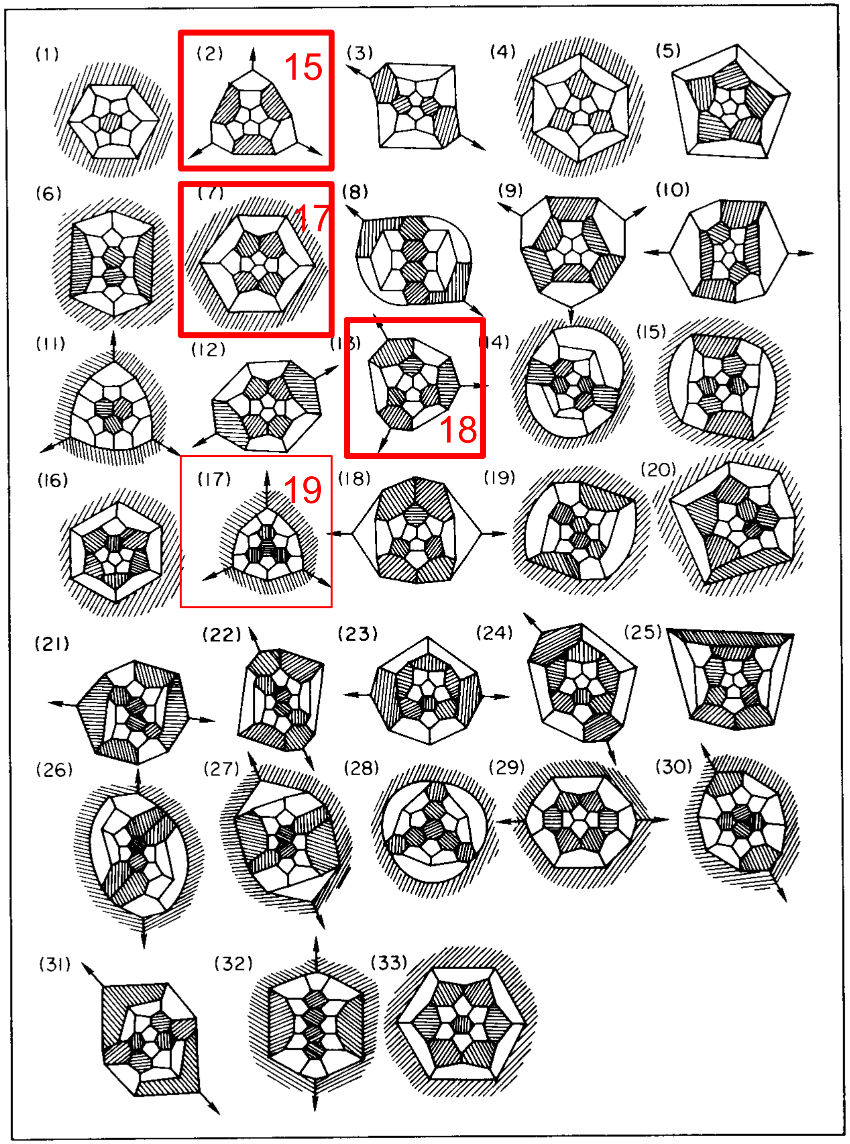


FIG. 3. Polyhedral structures that have twelve pentagonal and at most eight hexagonal faces and at whose vertices three edges meet. Shown in distorted representation. These structures are the ones enumerated in Table 1. However, the structures having nine and ten hexagonal faces are omitted in this figure and Table 2, because they are not important in the discussion below according to Appendix 2. The three structures in Fig. 1 correspond to (4), (32) and (33) in this figure.

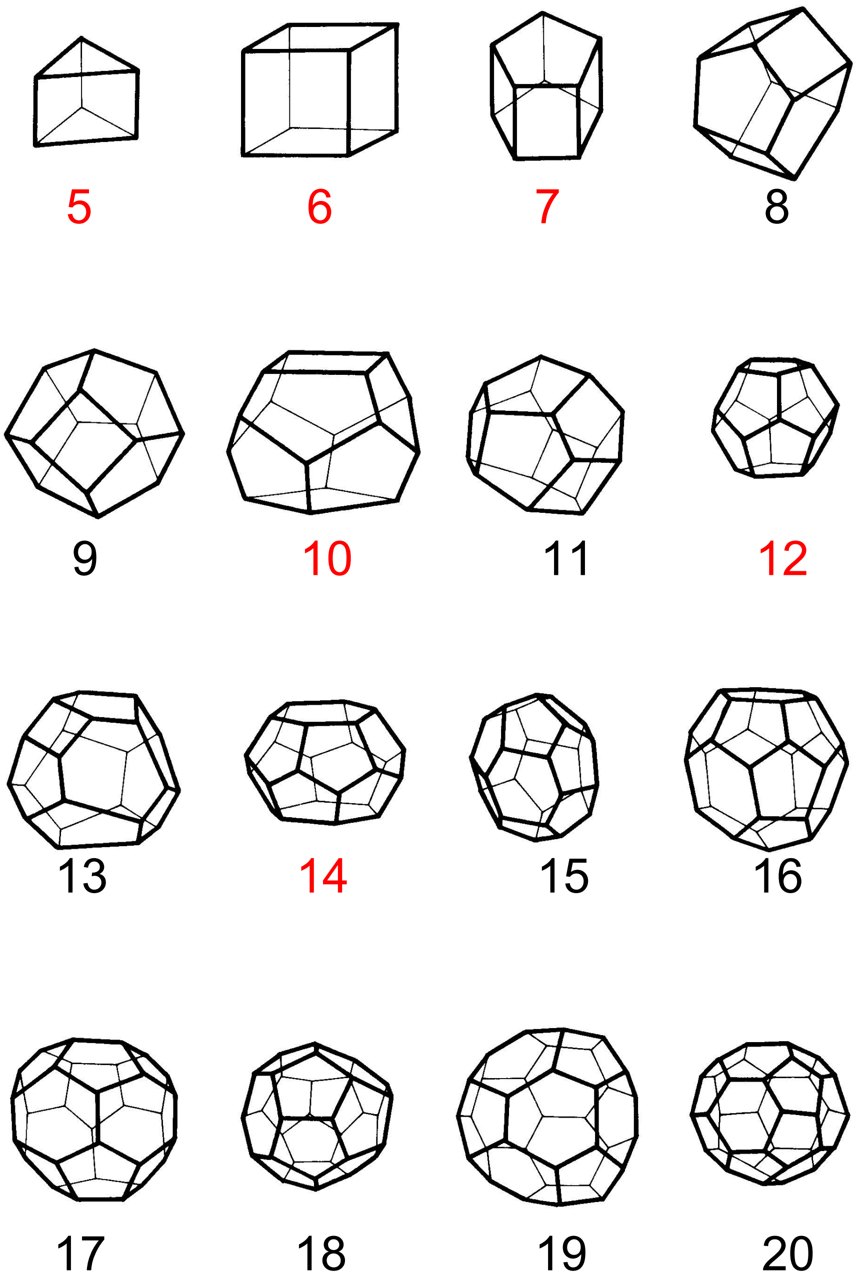
Theoretical clathrin baskets by Katsura

Minimum covering:

n = 15, 17, 18, 19

T.T. & Zs. Gáspár 1991

The result for n = 19 was improved in 1994 by R.H. Hardin, N.J.A. Sloane & W.D. Smith.



Solutions to the covering problem, $n \leq 20$

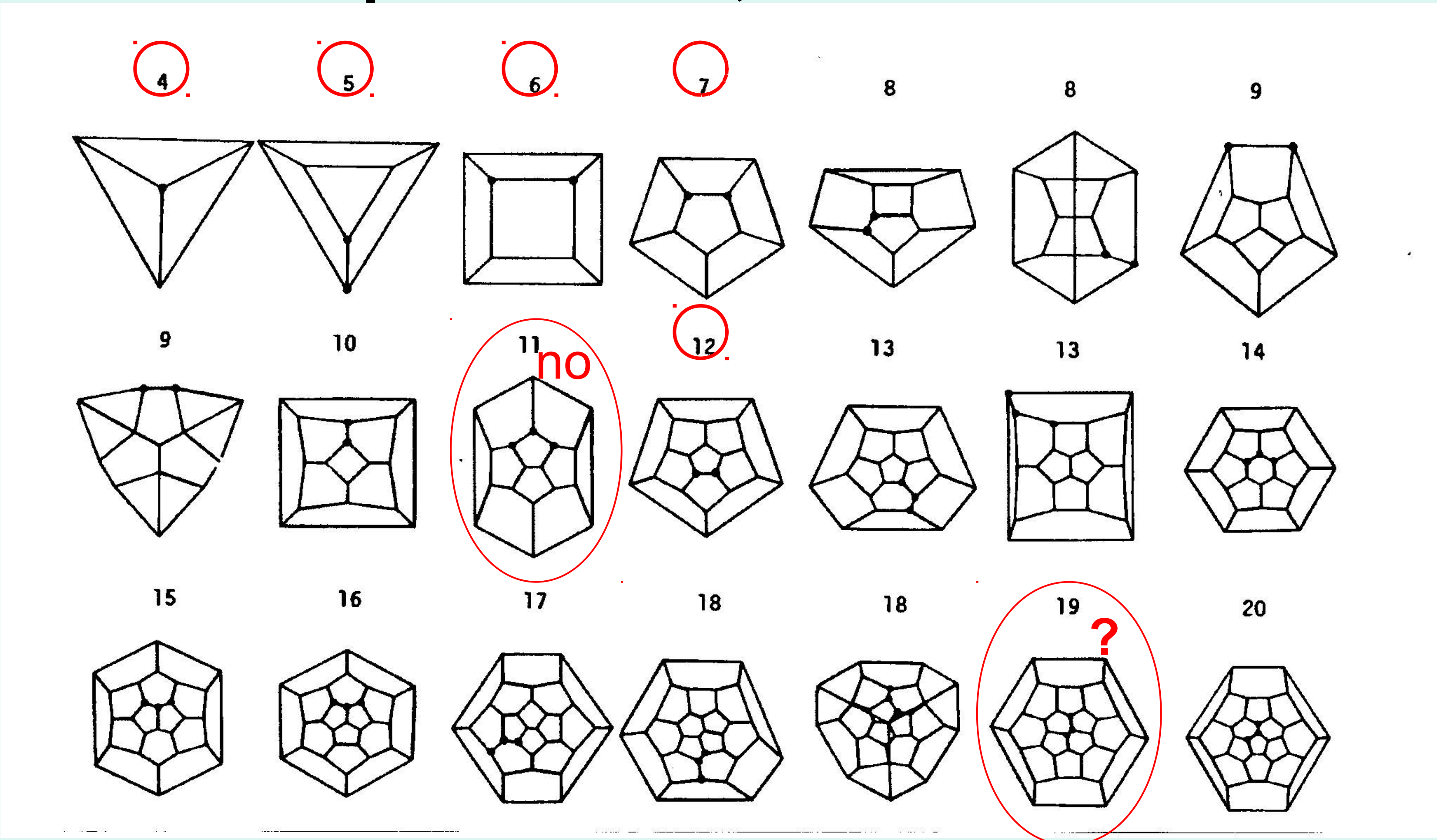
n = 2, 3, 4, 6, 12 L. Fejes Tóth
n = 5, 7, 8 K. Schütte
n = 9 E. Jucovič
n = 10, 14, 32 G. Fejes Tóth
n = 11, 13, 15, 16, 17, 18, 19, 20 T.T. & Zs. Gáspár
n = 19, 21 to 130 R.H. Hardin, N.J.A. Sloane & W.D. Smith

ISOPARAMETRIC PROBLEM FOR POLYHEDRA

Isoperimetric problem for polyhedra

Among polyhedra with given surface area and a given number of faces *n*, which has the maximum volume?

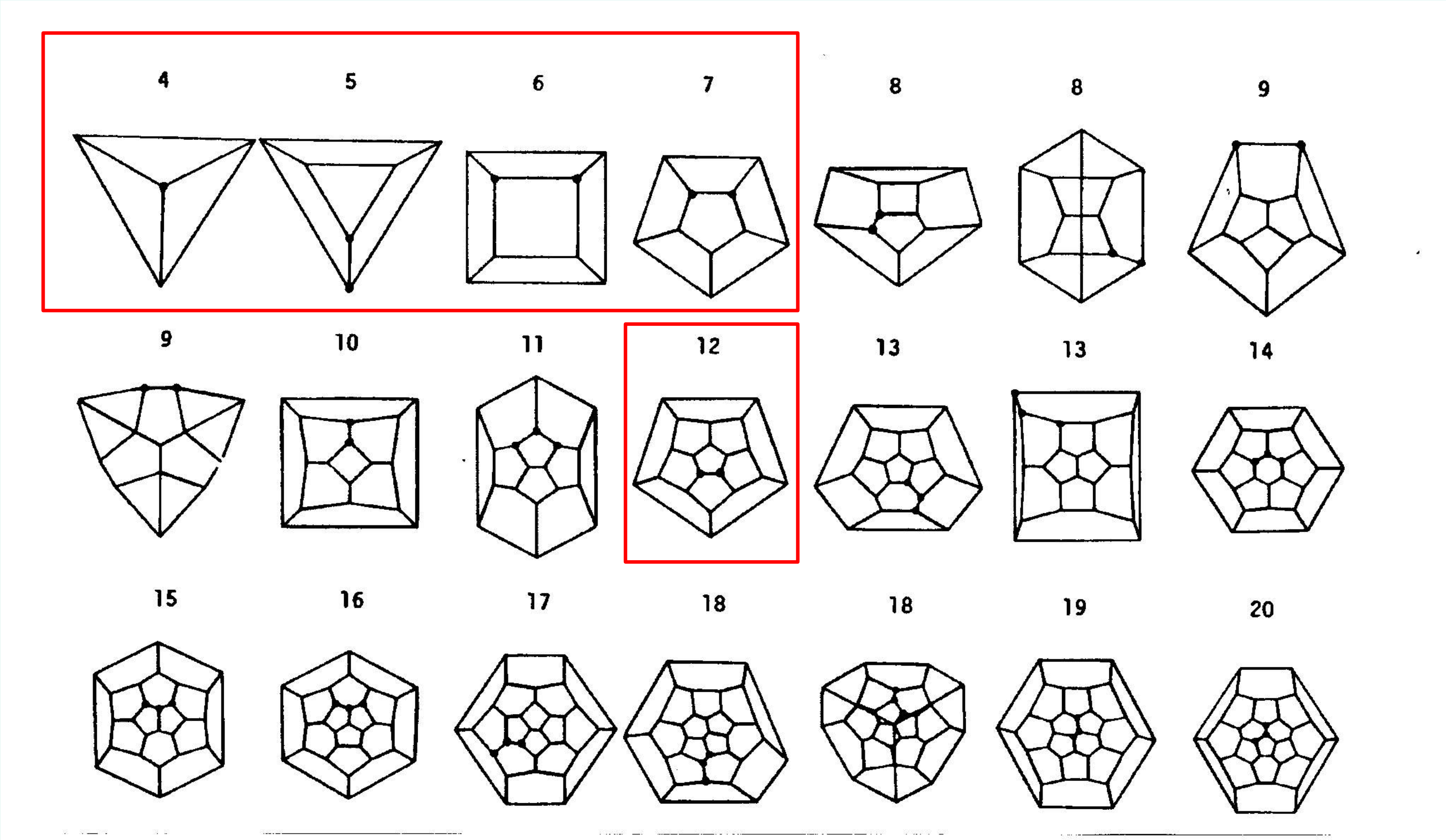
Solutions to the isoperimetric problem, $n \leq 20$



Conjectured solutions by M. Goldberg 1935, A. Schoen 1986

The edge graphs of the polyhedra are identical to those of covering except for n = 11. For n = 19, it is not known.

Identical solutions for both covering and isoperimetric problems



T.T., Zs. Gáspár, K. Hincz 2013

Identical solutions for both covering and isoperimetric problems, n = 32

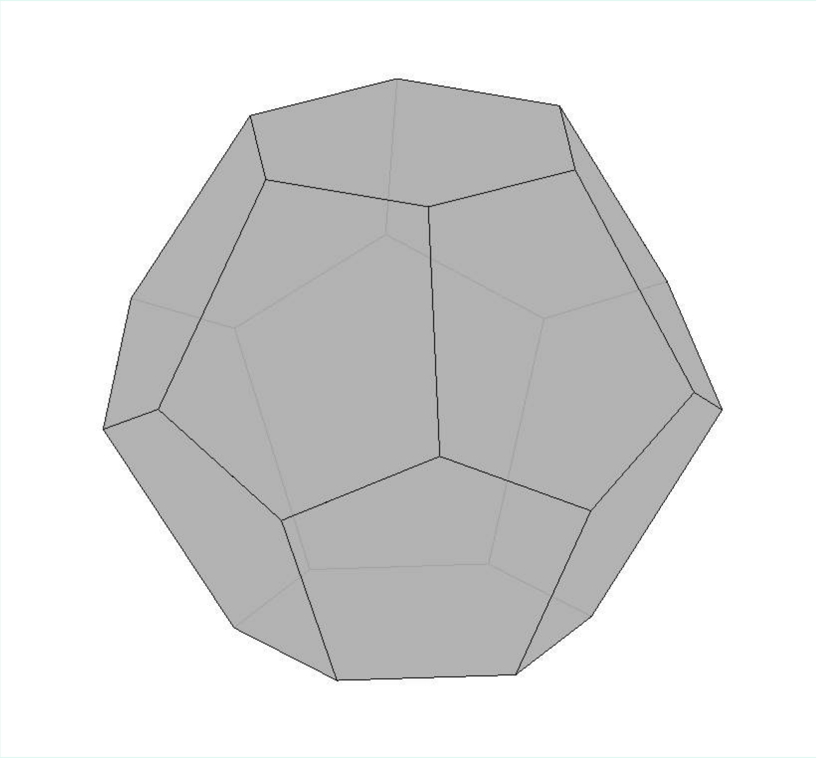
Minimum circle covering

Turned ivory, around 1600, Grünes Gewölbe, Dresden

Roundest soccer ball, the Hyperball

Solutions to covering and isoperimetric problems, *n* = 14

The two solutions are equal under O_h symmetry constraint



Only the edge graphs of the two solutions are equal if there is no symmetry constraint

Conclusions

- Soap bubbles in foam, and coated vesicles provided useful ideas to construct locally optimal coverings of the sphere with equal circles.
- The isoperimetric problem for polyhedra, and the problem of minimum covering the sphere with equal circles are related problems, since in both cases the obtained polyhedra have an insphere.

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