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Quantifying Contagion Risk in Funding Markets: An Application to Stress-Testing

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2. Model
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   b. Contagious runs
3. The Macro-Financial Risk Assessment Framework
   a. Stress-testing in Canada
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*“In **hectic times**, when fear dominates, the **absence of transparency** fosters **herd behavior** and **amplifies** considerably **the initial shock** that triggered the turbulences.”*

Jean-Claude Trichet, Salzburg Seminar 2007

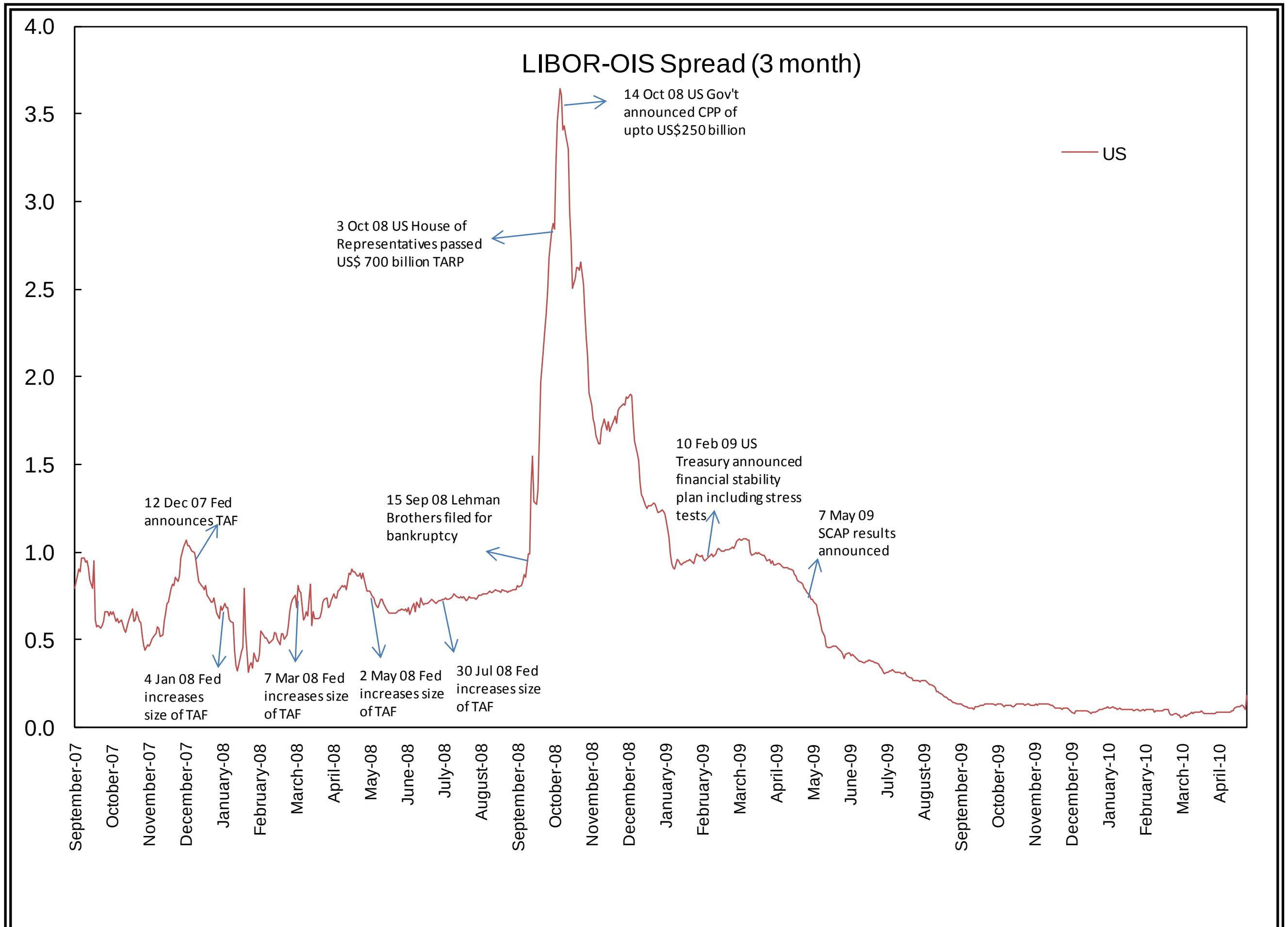
1. Introduction: “Bad news”

- Sub-prime crisis – put in motion by the announcement from BNP Paribas on August 9th, 2007
- News triggered general market anxiety about the extent of banks’ exposures to sub-prime mortgages and solvency
- Balance-sheet opaqueness – funding conditions stressed

1. Introduction: “Good news”

- On the flip side – good news can have a positive impact
- Supervisory Capital Assessment Program stress-test results
  - Credible results about prospective losses at banks
  - Prompted successful recapitalization
  - Helped restore confidence in the banking system

1. Introduction: “Good news”



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1. Introduction: Our paper

- Financial contagion – the reaction of market participants to news about financial distress at other financial institutions
- Opaque balance sheets and limited information – precipitate bank runs, sweeping away solvent and insolvent banks alike
- Tractable model-based stress-testing framework to
  - Assesses solvency, liquidity and information contagion risk
  - Investigate the role of prior beliefs in the unfolding events

1. Introduction: Our contribution

- Generalize the information-contagion results of Manz (2010) to an N-banks world and show the uniqueness and existence of an equilibrium in that context.

- Demonstrate and quantify the contribution of information contagion to systemic risk using the Macro-Financial Risk Assessment Framework (MFRAF) *

* "At the frontier of systemic risk stress-testing" – IMF (2014)

2. Model

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2. Model: Environment

- Three periods – initial (0), interim (1) and final (2)
- N leveraged financial institutions, or “banks”
- Assets: cash (M) and risky assets (I) with returns equal to $R \times I$
- Liabilities: equity (E), short-term (ST) and long-term (LT) loans
- Each bank is funded by a distinct continuum of creditors who offer both type of loans

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2. Model: Environment

- In period 0, Nature determines banks' balance-sheet composition
- Each bank can incur losses on its risky assets:
  - Interim date losses: $P^{(1)}$
  - Final date losses: $P^{(2)}$
- Final date loss cumulative distribution $F_2(.)$ and support $\left[\underline{P}^{(2)}, \bar{P}^{(2)}\right]$.
- Interim date:
  - Creditors decide to roll-over maturing short-term claims, based on current and future expected losses

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2. Model: Timeline

Date 0

- Nature assigns creditors to each bank and determines the starting balance sheets

Date 1

- Creditors of bank *k* receive private signals on intermediate losses, and decide on whether to roll over claims

Date 2

- Final period losses are realized and liquid banks face insolvency risk

2. Model: Risks

- Each bank face two risks
  - Solvency risk: A bank's capital is insufficient to cover losses incurred at the interim date, or expected cumulative losses at the final date
  - Funding liquidity / Rollover risk: Concerns over a bank's potential insolvency in the final period will reverberate amongst its creditors, leading some to a withdraw their claims.

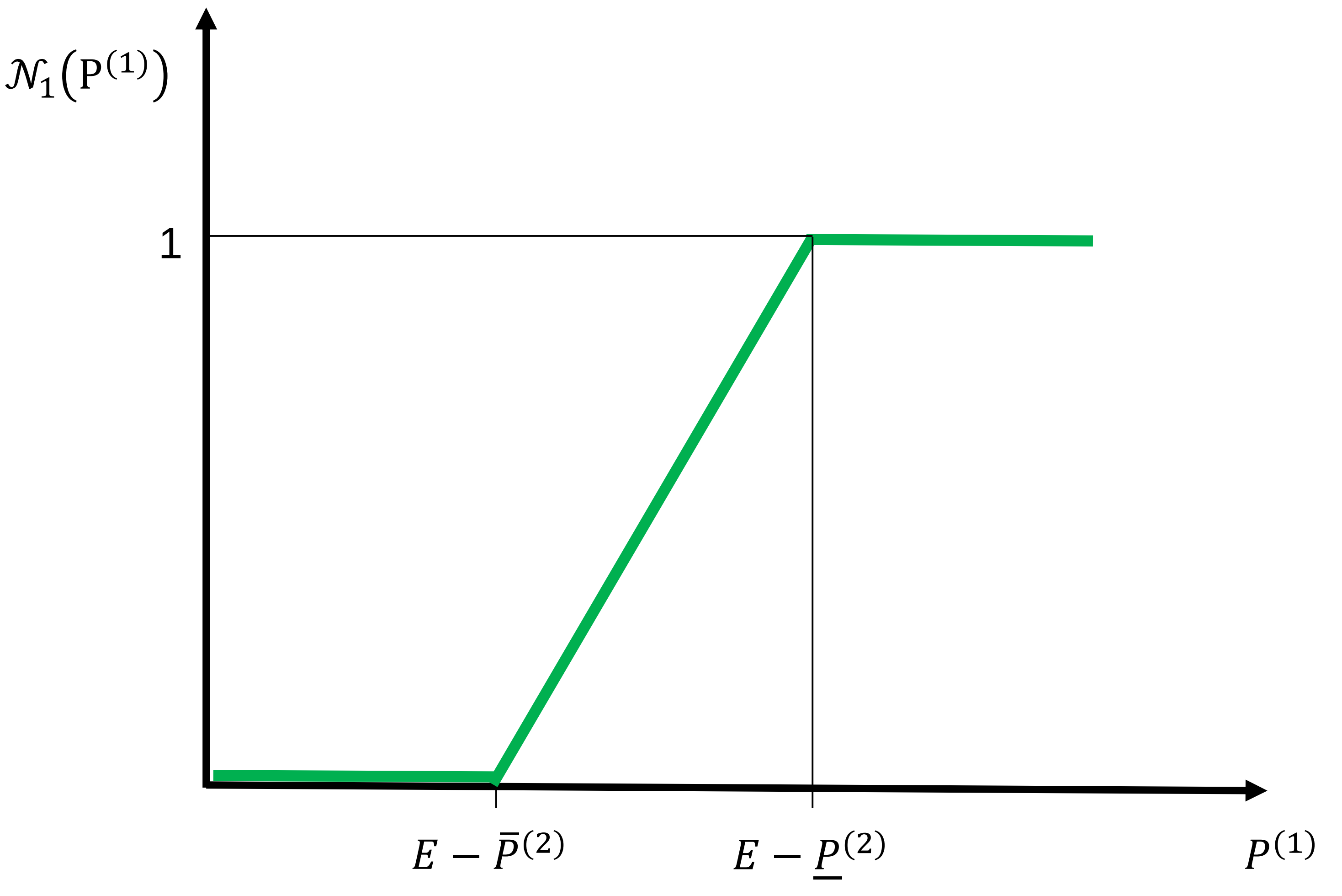
2. Model: Interim solvency risk

- Following $P^{(1)}$, a bank will be insolvent at the final date with probability:

$$\mathcal{N}_1\left(\mathrm{P}^{(1)}\right) = \mathrm{Prob}\left(\tilde{P}^{(2)} > E - P^{(1)}\right)$$

$$= \begin{cases} 1, & \text{if} \quad P^{(1)} > E - \underline{P}^{(2)} \\ 1 - F_2\left(E - P^{(1)}\right), & \text{if} \quad E - \bar{P}^{(2)} < P^{(1)} \leq E - \underline{P}^{(2)} \\ 0, & \text{if} \quad P^{(1)} \leq E - \bar{P}^{(2)} \end{cases}$$

2. Model: Interim solvency risk



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2.a Rollover risk

- Modeled as binary action simultaneous move coordination game
- Creditors estimate their bank's recourse to liquidity:
  - Liquid Assets
  - Funds raised by selling available risky assets
- A key quantity in the model is the discount rate $\psi < 1$, i.e., cash raised from liquidating a unit of risky asset

2.a Rollover risk

- **Illiquidity condition:** a bank fails if the fraction of creditors who withdraw (ℓ) is greater than the banks' recourse to liquidity, i.e.,

$$\ell \times ST > M + \psi \times \left(R \times I - P^{(1)}\right),$$

where $\bar{\psi}$ is the expected fire-sale price for the bank's illiquid assets

Balance Sheet Liquidity: $$\lambda \equiv \frac{M + \psi \times \left(R \times I - P^{(1)}\right)}{ST}$$

2.a Rollover risk

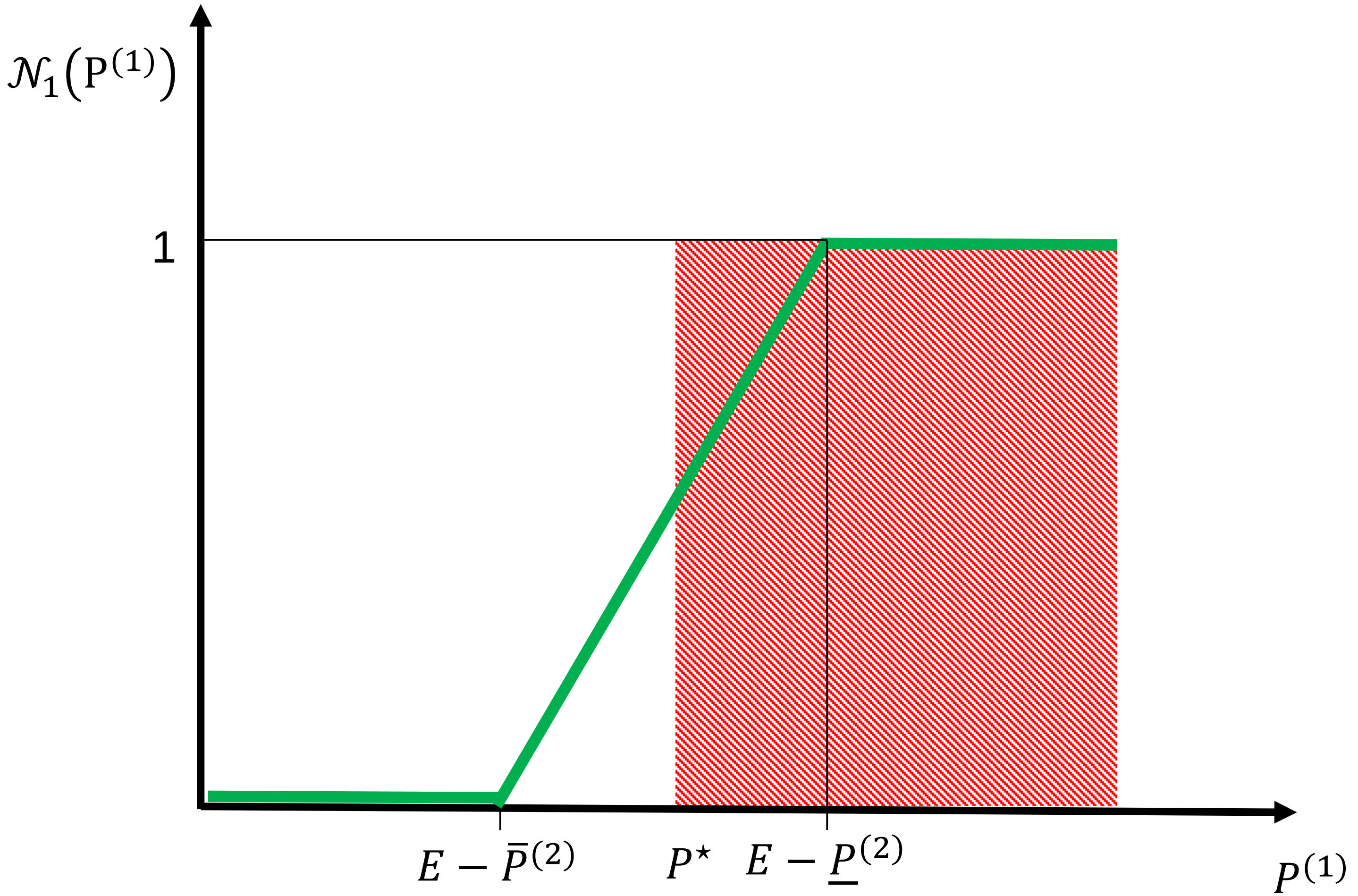
	$\ell \leq \lambda$	$\ell > \lambda$
Rollover	r^S	0
Withdraw	r^{F}	r^{F}

2.a Rollover risk

- Solve using the global games paradigm
- Creditors use threshold strategies
  - rollover if $P^{(1)} < P^{\star}$
  - foreclose otherwise
- Bayes-Nash Equilibrium – $P^{\star}$ solved from FPE

$$\lambda(P^{\star}) \times \mathrm{Prob}\big(E_0 - P^{\star} - P^{(2)} > 0\big) \times r^S = r^F$$

2.a Rollover risk



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2.b Contagious runs

- At the initial date, Nature draws $\psi \in \{\psi_L, \psi_H\}$, but this is unobserved

- Creditors of bank k believe that the $\psi = \psi_H$ with probability w_k, while with probability $1 - w_k$, they believe that $\psi = \psi_L$

- The true value is realized only at the final date.

2.b Contagious runs

Date 0

- Nature assigns creditors to each bank and determines the starting balance sheets
- Nature determines initial beliefs of banks' creditors about discount rates

Date 1

- Creditors of bank *k* receive private signals on intermediate losses, and decide on whether to roll over claims
- Creditors of bank *i* observe the outcome of bank *j*, update their beliefs and possibly revise their decisions

Date 2

- Discount rate and final period losses are realized and liquid banks face insolvency risk

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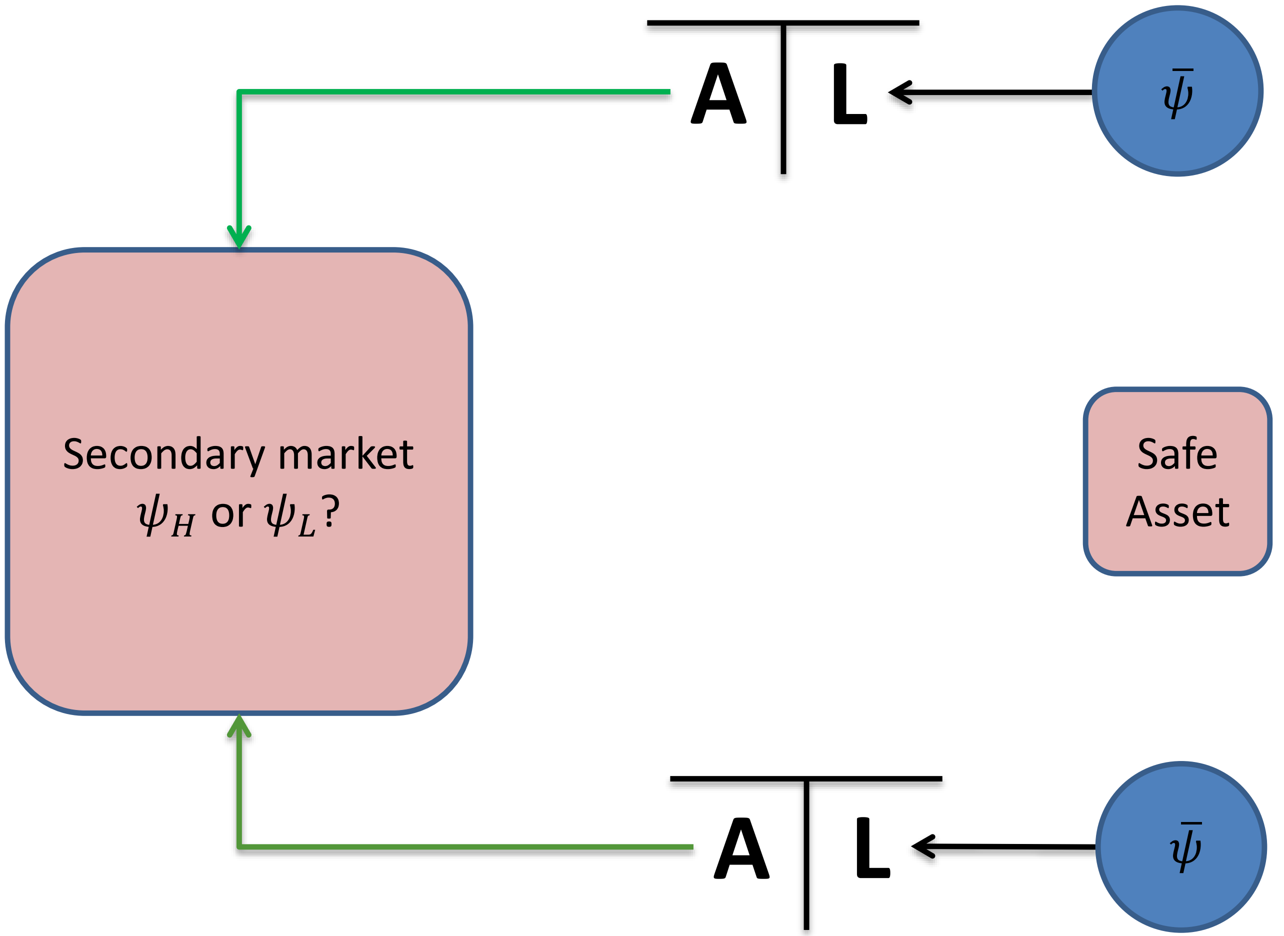
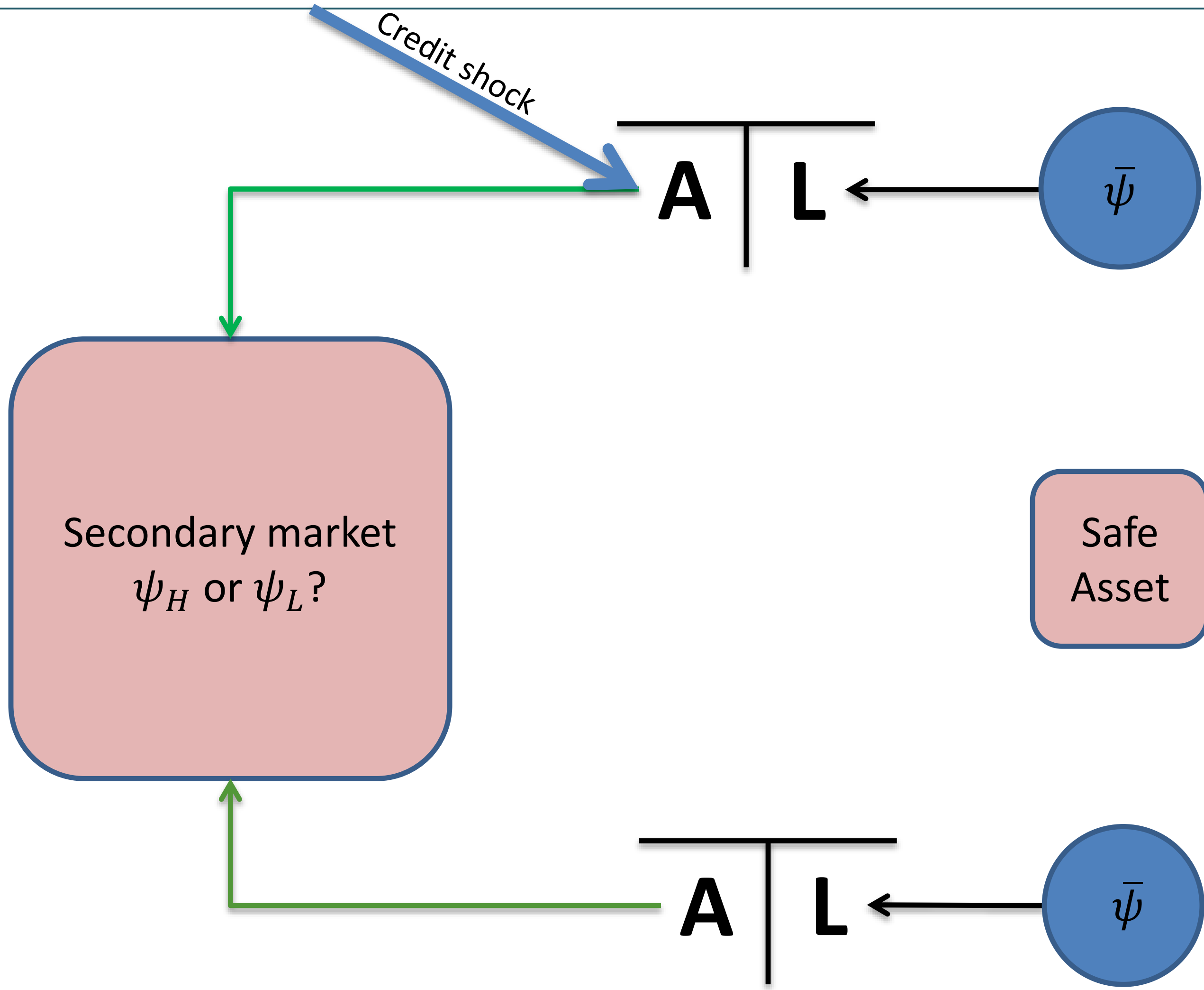
2.b Contagious runs



2.b Contagious runs



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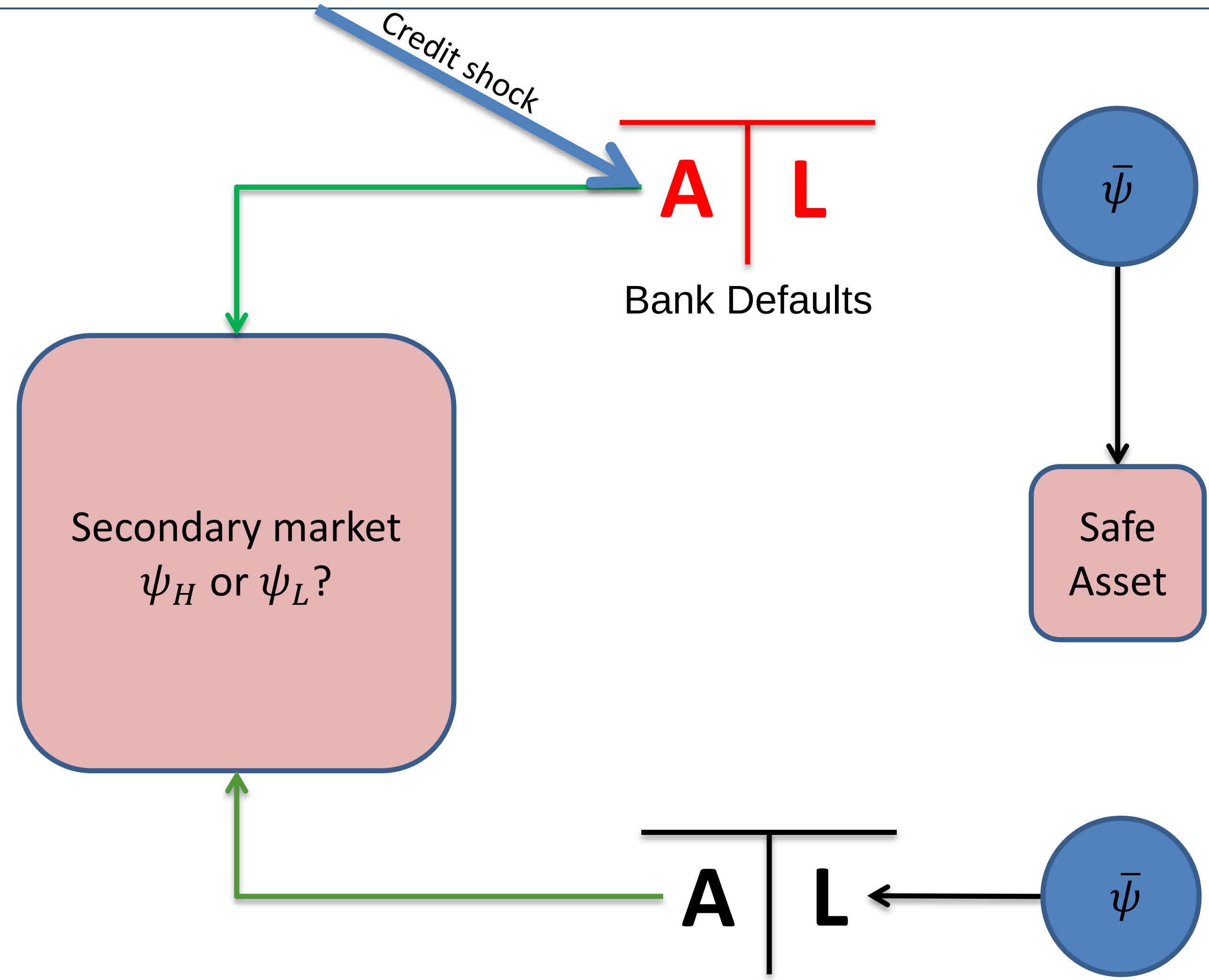

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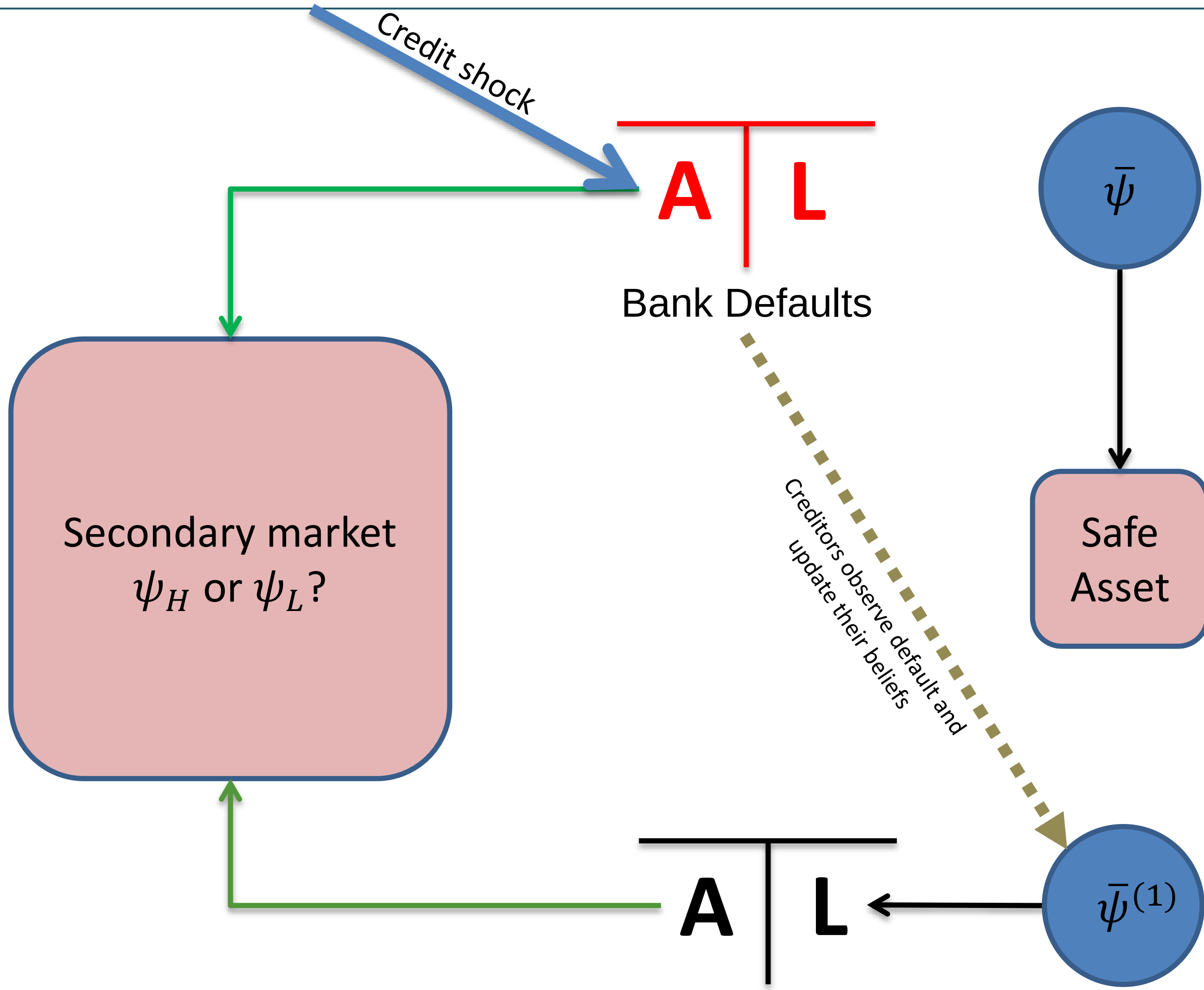

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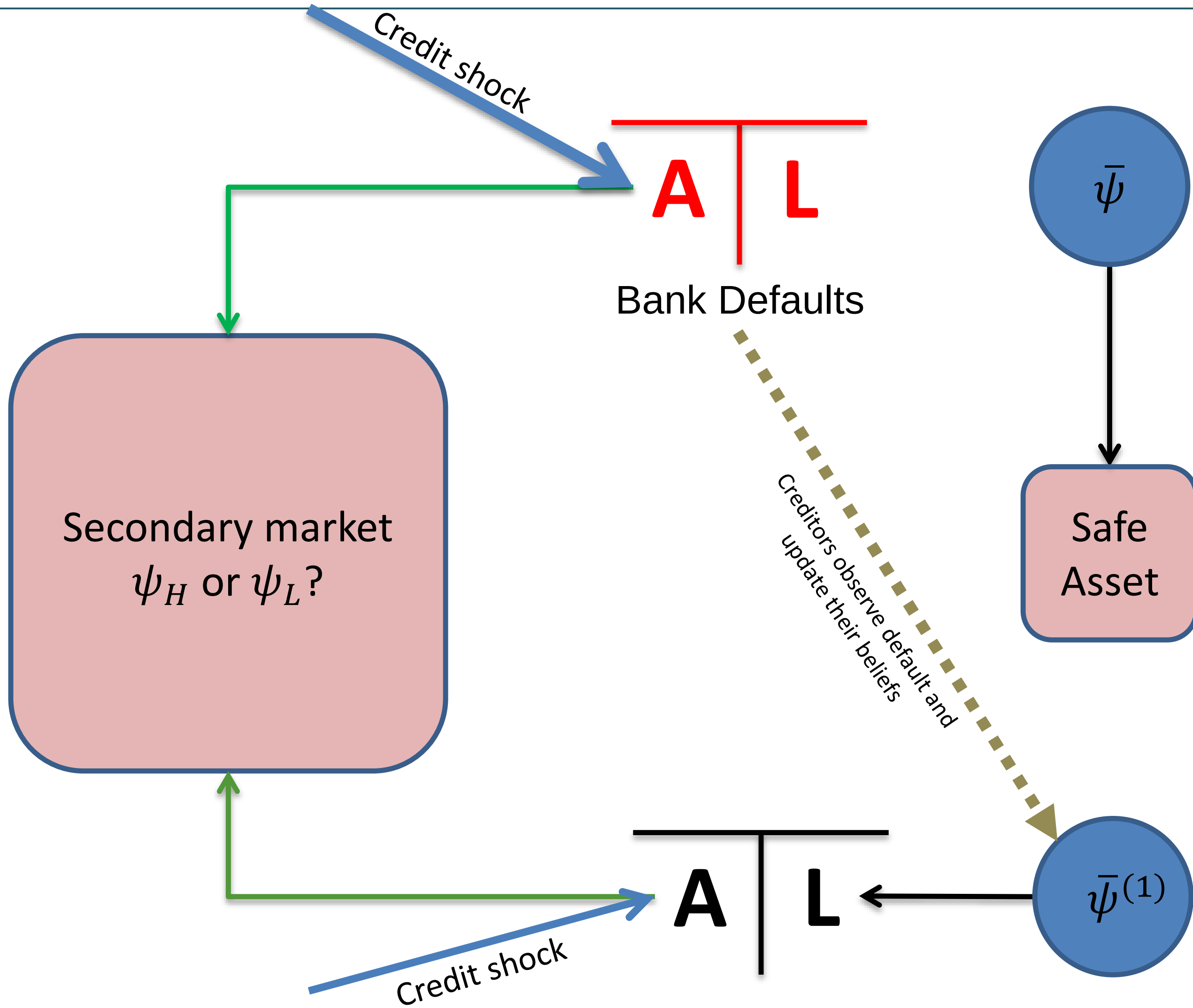

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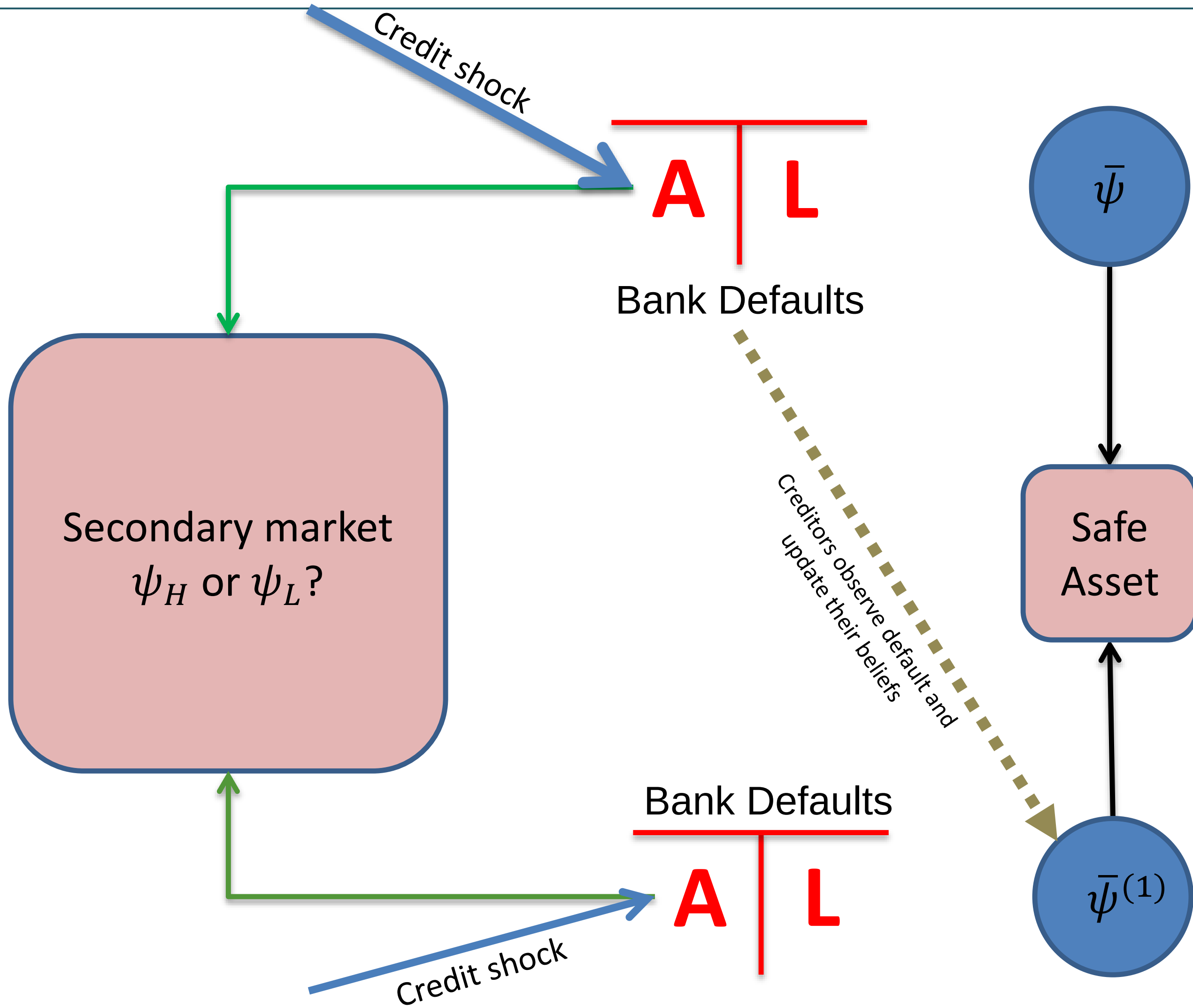

2.b Contagious runs: Bayesian updating

- Define w_j to be the subjective belief held by the creditors of bank j that $\psi = \psi_H$, and $\eta_k \in \{0,1\}$ as an indictor for whether bank k has defaulted (1), or not (0), and i as the iteration-step

$$w_j^{(i+1)} = Prob\left(\psi = \psi_H \mid \left\{\eta_k^{(i)}\right\}_{k \neq j}\right)$$

$$= \frac{Prob\left(\eta_1^{(i)} \mid \left\{\eta_k^{(i)}\right\}_{k \neq j,1}, \psi_H\right) \times \cdots \times Prob\left(\eta_{N-1}^{(i)} \mid \eta_N^{(i)}, \psi_H\right) \times Prob\left(\eta_N^{(i)} \mid \psi_H\right) \times Prob(\psi = \psi_H)}{Prob\left(\left\{\eta_k^{(i)}\right\}_{k \neq j}\right)}$$

$$= Prob(\psi = \psi_H) \times \prod_{k \neq j} \frac{Prob\left(\eta_k^{(i)} \mid \psi_H\right)}{Prob\left(\eta_k^{(i)}\right)}$$

Proposition 1

A new failure decreases the creditors of each surviving banks' belief that the risky asset has the high value. The decrease in beliefs in any given round is an increasing function of the number of new failures in that round.

Proposition 2

The rollover game for information contagion will terminate after, at most, N iterations.

3. Macro-Financial Risk Assessment Framework

3.a. Stress-testing in Canada

- Annual exercise conducted jointly by the BoC and OSFI involving big-6 Canadian banks
- **Objective:** To assess the resilience of the financial system to extreme but plausible shocks
- MST scenario
- Bottom-up stress test exercise:
  - Bank's apply MST scenario to their balance sheet
  - Focuses on solvency risk only
- Top-down stress test exercise:
  - MFRAF

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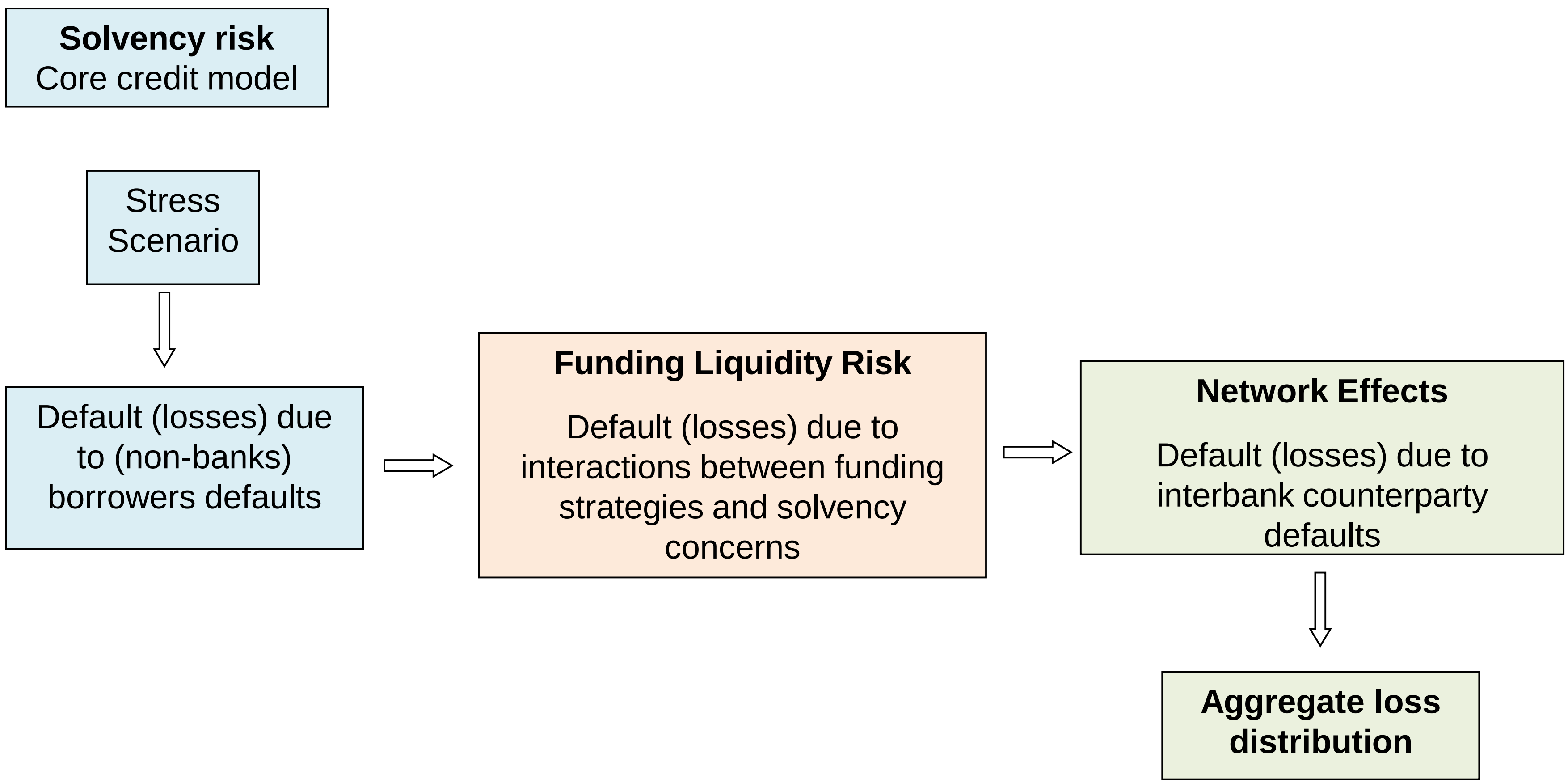
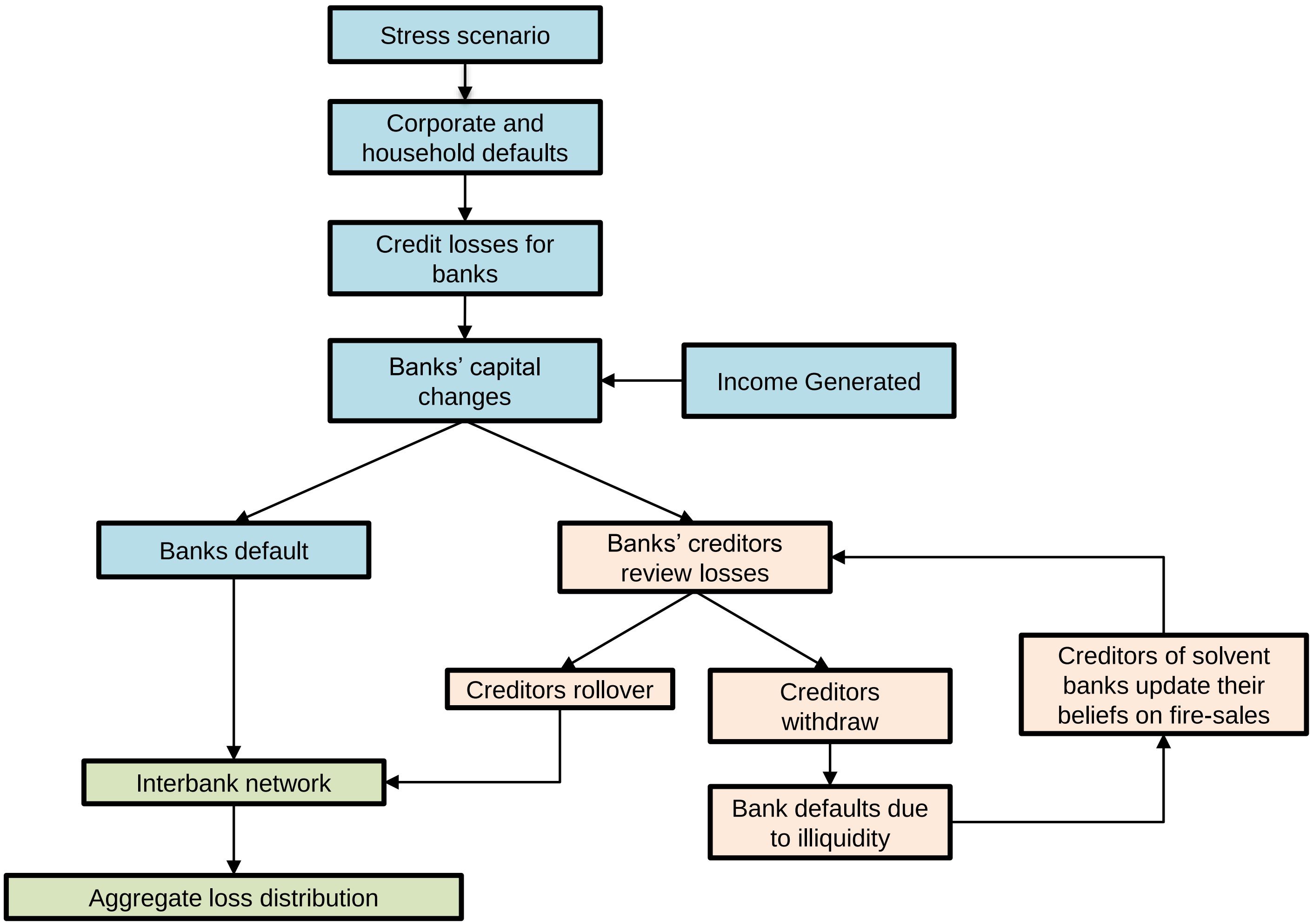
3.b MFRAF: Framework



3.b MFRAF: Framework



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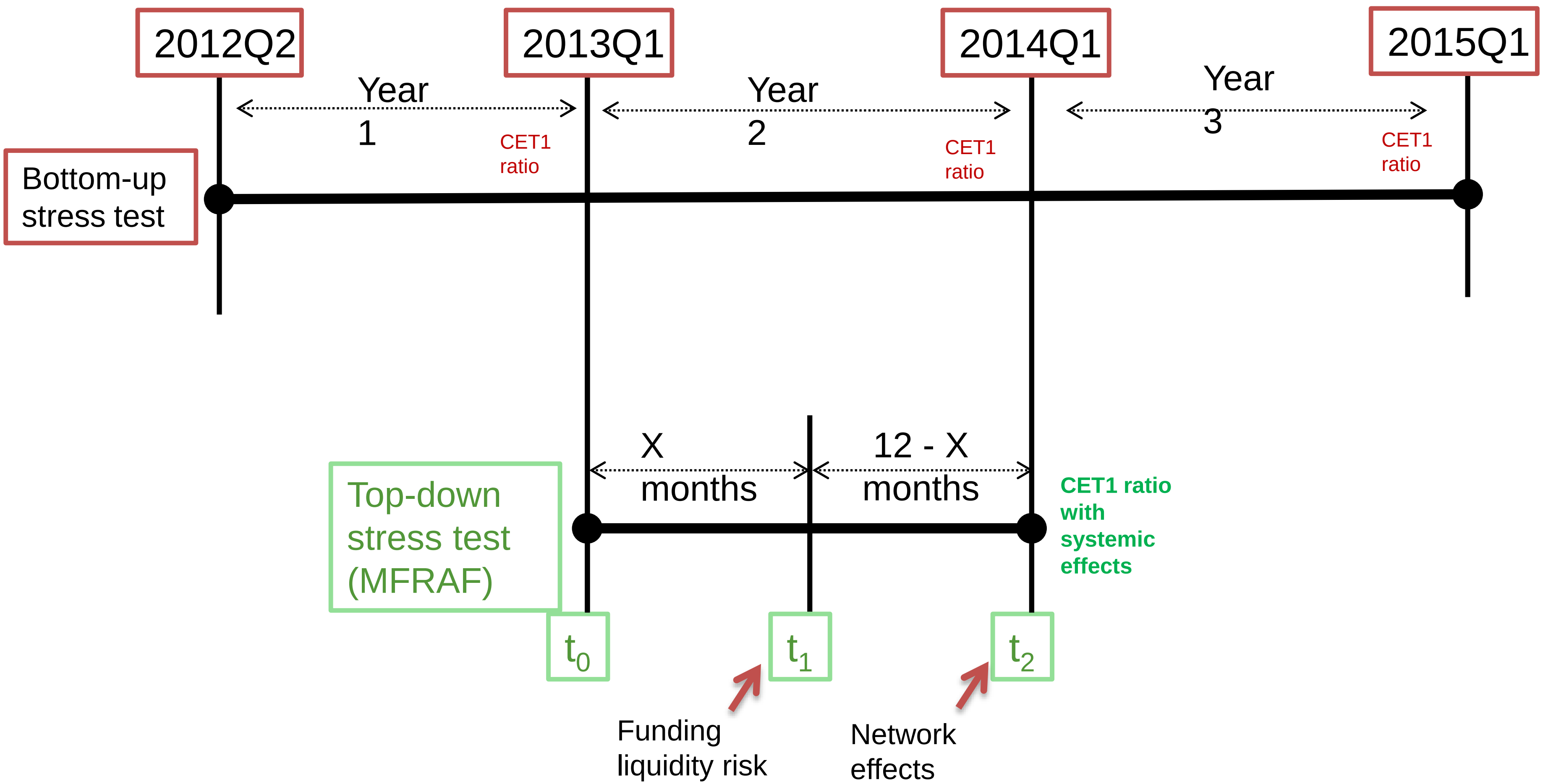
3.b MFRAF: Timeline



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3.b MFRAF: Solvency risk

- Banks' loan portfolios subject to credit risk across different sector

$$\text{Expected Losses} \\ = \text{Probability of Default} \times \text{Loss Given Default} \times \text{Exposure at Default}$$

  - PDs (distribution) – function of macro-variables.
  - LGDs – judgement based, e.g., from bottom-up exercises
  - EADs – banks' regulatory reported values

- Derive **annual loss distributions** for each sector and for each bank

3.b MFRAF: Solvency risk

- Each realization of the expected annual losses, $P^{(E)}$, must be translated into the time structure of MFRAF

- Losses $P^{(1)}$ realized at date t_1 (interim period)

- Losses $P^{(2)}$ realized at date t_2 (final period)

$$P^{(1)} = \frac{P^{(E)}}{12/X}, \text{and } \; P^{(2)} = P^{(E)} \times \left(1 - \frac{1}{12/X}\right)$$

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3.b MFRAF: Liquidity risk

- At the interim date, t_1, following the realization of the $P^{(1)}$ losses, a bank's creditors may decide to run
- Runs may occur due to:
  - Concerns over the bank's **future solvency**;
  - **Low liquidity**, relative to it's wholesale funding
- Use the insights from the theory of rollover risk and contagious runs

3.c MFRAF: Results

- Leverage, capital ratios and riskiness and distribution of loan portfolios - same for all banks, and calibrated at their average values for the Big six Canadian banks as of 2012Q4

Bank	$\bar{\lambda}$	Solvency Risk	Liquidity Risk	Contagion Risk	Total Risk
1	0.99	47.0	22.9	0.0	69.9
2	1.43	47.0	0.0	0.0	47.0
3	0.76	47.0	23.0	0.6	70.6
4	1.07	47.0	0.0	19.2	66.2
5	1.39	47.0	0.0	0.0	47.0
6	0.86	47.0	22.2	0.8	70.0

3.c MFRAF: Results

- Comparative static exercise – change ratio of maturing liabilities to total assets for banks 2 and 5

Bank	$\bar{\lambda}$	Solvency Risk	Liquidity Risk	Contagion Risk	Total Risk
1	0.99	47.0	22.9	0.0	69.9
2	1.07	47.0	0.0	22.6	69.6
3	0.76	47.0	23.0	0.6	70.6
4	1.07	47.0	0.0	19.2	66.2
5	1.07	47.0	0.0	19.7	66.7
6	0.86	47.0	22.2	0.8	70.0

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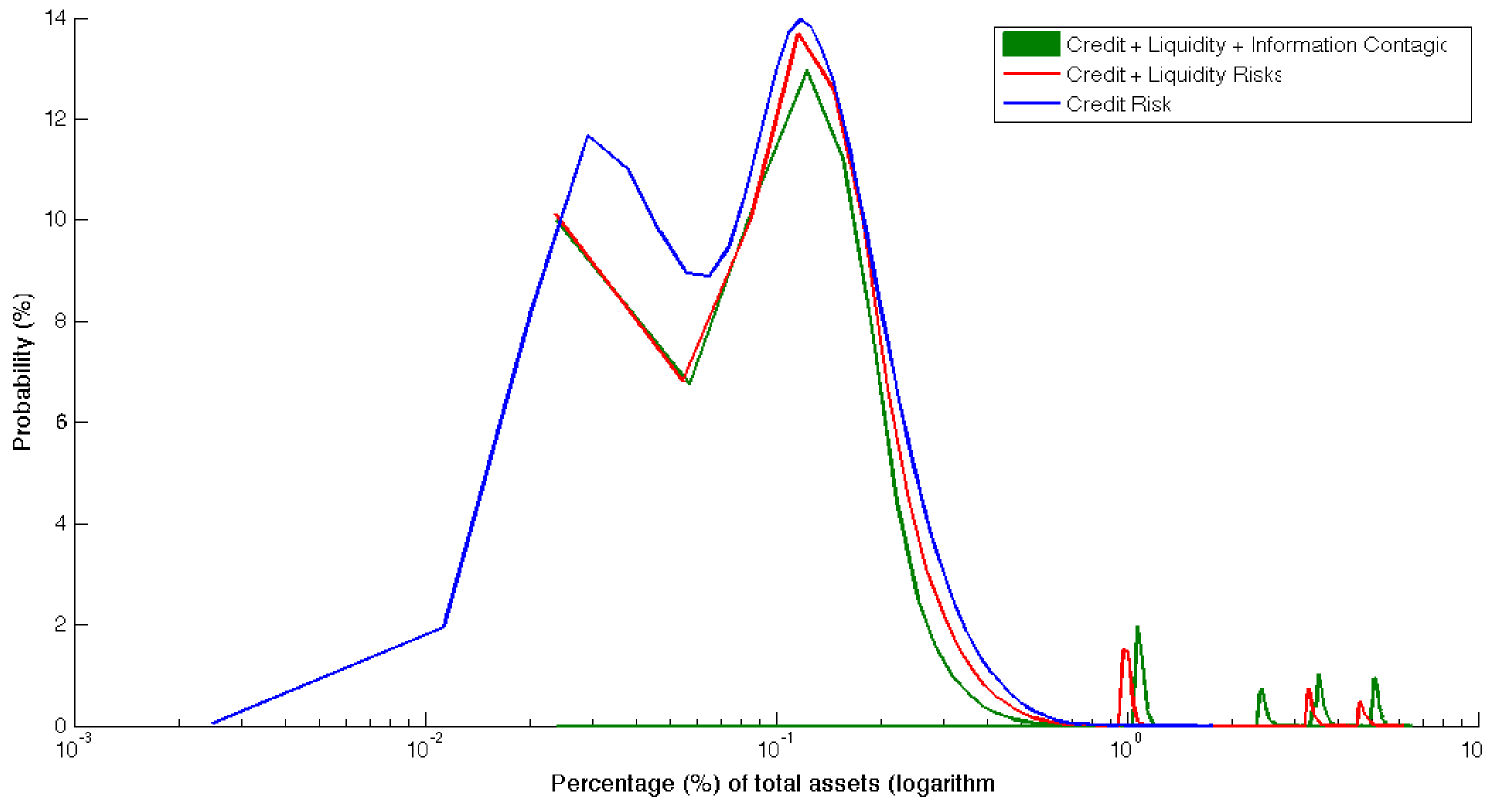
3.c MFRAF: Results



3.d MFRAF: Use in Policy

- Consistency check for bottom-up results
- Considers impact of second-round effects over and above the (solvency only) bottom-up stress-test
- Quantifies liquidity assistance required to avoid runs
- Investigate how heterogeneous creditor beliefs can exacerbate risks and the role of communication

4. Conclusions

4. Conclusions

- MFRAF is a top-down stress testing tool that investigates the interactions between solvency and liquidity risk.
- Results depend starting capital ratios and balance sheet liquidities.
- Calibrating prices is very much an art form, and ideas for a more robust modeling would be very welcome.

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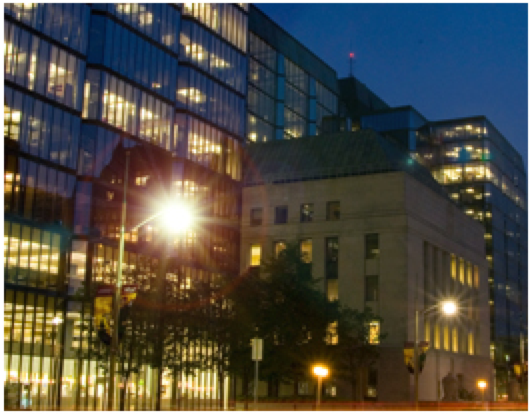
4. Conclusions: Model Improvements – Key priorities

- Feedback effects to the real economy
  - TVAR with endogenous Financial Stress Index (FSI) to generate stress scenarios
  - Link FSI to outputs from MFRAF (e.g., via losses).

- RWA model to account for impact of liquidity risk and network effects.

- Link market liquidity (ψ parameters) with funding liquidity risk, i.e., endogenous relationship.

Thank you!



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Appendix: Calibrating MFRAF

3 Calibrating MFRAF

- Banks reported their holdings of liquid and illiquid assets using the Net Cumulative Cash Flow (NCCF) definitions
- Liquid assets have to be unencumbered and eligible for central bank open market operations:
  - Cash and deposit accounts at the BoC
  - Government securities (Canada, U.S., and Euro Area)
  - Other eligible securities (e.g. BAs and NHA-MBS)

3. Calibrating MFRAF

Variable	Description	Source for calibration
I_0	Dollar value of illiquid assets	NCCF report
ψ_H	Liquidation value of assets in the "high" state	Judgement on haircuts
ψ_L	Liquidation value of assets in the "low" state	Judgement on haircuts
M_0	Dollar value of liquid assets	NCCF report
S_0	Cumulative short term liabilities that come to maturity in t_1	NCCF report
RWA	Risk weighted assets (CET1 Basel III)	Provided by the banks
$INCOME$	Operating income (internally generated capital)	Satellite models
κ	Bank's starting capital levels (CET1 Basel III)	Provided by the banks
τ	Minimum threshold level for bank's capital ratio (7% or 4.5%).	
X	Interbank network	Regulatory filings

3. Assumptions on discount rates

Instrument	State H	State L
Deposits with banks		
Other Securities		
Other government		
Mortgage Backed Securities		
Asset Backed Securities		
Corporate CP		
Corporate bonds		
Equities		
Precious Metals		
Other commodities		

3. Assumptions on discount rates

Instrument	State H	State L
Loans		
Residential mortgages - insured		
Residential mortgages - uninsured		
Personal loans		
Credit cards		
Business and government loans		
Customers' liabilities under BAs		
Swapped Intra-bank Loans		
Call Loans		
Reverse Repurchase Agreements		
Securities borrowed		
Derivatives related amounts		
Other Assets		

3. Starting capital level (CET1 Basel III)

- "Front-load" income generated over the 1-year MFRAF horizon onto the starting capital level, i.e.,

$$E_0 = \kappa \times RWA_0 + Income$$

- To determine the default threshold, we look at the level of capital in excess of the regulatory minimum,

$$E_0 = \kappa \times RWA_0 + Income - \tau \times RWA_0$$

3. Accounting for losses

- Credit risk losses

$$P^{(1)} + P^{(2)}$$

- Losses following a bank run

z percent of $\tau \times RWA_0$

- Losses after default due to network contagion

endogenous clearing